Modellen en algoritmen
voor het cyclisch inventory routing probleem

Models and Algorithms for the Cyclic Inventory Routing Problem

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Samenvatting

Probleembeschrijving

De basisidee van 'Supply Chain Management' is om de verschillende stappen in een bevoorradingsketen te laten samenwerken. Door de beslissingen die genomen worden bij het plannen van activiteiten op elkaar te gaan afstemmen, kan een beter globaal resultaat bekomen worden. Een mooi voorbeeld van integratie van verschillende beslissingsniveaus vinden we bij 'Vendor Managed Inventory' (VMI), waarin een leverancier gaat samenwerken met de klanten die hij belevert. In een VMI-strategie is het de leverancier die de verantwoordelijkheid op zich neemt om de voorraad van zijn klanten te onderhouden, zodat de klanten geen bestellingen meer hoeven te plaatsen. De leverancier kan op die manier zelf kiezen hoe vaak, op welke tijdstippen en in welke hoeveelheden hij bij zijn verschillende klanten gaat beleveren. Die vrijheid geeft hem de kans om efficiëntere routes samen te stellen voor zijn voertuigen. Het optimalisatieprobleem waarmee de leverancier geconfronteerd wordt, namelijk tegelijkertijd de te leveren hoeveelheden bepalen en routes voor de voertuigen ontwerpen om alle klanten te bezoeken, staat bekend als het 'Inventory Routing' probleem (IRP).

In dit proefschrift wordt het IRP bestudeerd voor de gevallen waarin het verbruiksniveau bij de klanten constant blijft. In de literatuur worden een aantal methodes voorgesteld voor het IRP met constante vraagpatronen. Deze methodes beschouwen alle een oneindige tijdshorizon en stellen cyclische oplossingen voor. Het is echter zo dat de bestaande modellen en oplossingsmethoden in de literatuur een aantal gebreken vertonen. Het grootste gebrek is dat er vanuit gegaan wordt dat er voor elke rit een voertuig beschikbaar is. Er wordt voorbijgegaan aan de mogelijkheid dat een voertuig meerdere ritten kan doen. De methoden zoeken afwegingen tussen transportkosten en voorraadkosten, maar houden geen rekening met de vaste voertuigkosten.

In dit proefschrift worden een nieuwe cyclische modelleringsaanpak en oplossingsmethode voorgesteld voor het Inventory Routing probleem met een deterministische, constante vraag bij de klanten. Daarin wordt een vaste voertuigkost wel mee in rekening genomen en wordt expliciet aangegeven dat één voertuig meerdere ritten kan maken. Het probleem van het toewijzen van ritten aan voertuigen wordt zo mee opgelost. De voorgestelde oplossingen maken dus niet zomaar een tweevoudige afweging tussen transportkosten en voorraadkosten, maar veeleer een drievoudige afweging tussen transportkosten, voorraadkosten en vaste voertuigkosten.

Modellering

Om de toewijzing van ritten aan voertuigen expliciet in het model in te bouwen, wordt een nieuw concept voorgesteld: een zgn. *distributiepatroon*. Dit is een cyclisch plan voor één enkel voertuig dat kan bestaan uit verschillende ritten die met eventueel verschillende frequenties herhaald worden.

Oplossingsmethode

Het cyclisch inventory routing probleem kan beschouwd worden als een combinatie van onderling afhankelijke deelproblemen. Hieronder staat een mogelijke top-down decompositie van het probleem.

1. Verdeel de klanten over één of meerdere voertuigen.
2. Verdeel de klanten van elk voertuig verder over één of meerdere ritten.
3. Bepaal de volgorde waarin de klanten binnen elk van de ritten bezocht worden.
4. Bepaal voor elk voertuig de frequenties van de verschillende ritten die het moet rijden.
5. Stel voor elk voertuig een tijdsschema op om de haalbaarheid van het vooropgestelde distributiepatroon na te gaan en om de kost per tijdseenheid te bepalen.

In dit proefschrift worden verschillende oplossingsmethodes voorgesteld die trachten goede kostenafwegingen en dus globaal kosten-efficiënte oplossingen te vinden. Eerst worden twee constructieve heuristieken besproken om toelaatbare oplossingen te genereren. De eerste is een sequentiële invoegheuristiek waarin de klanten één voor één in de oplossing worden ingevoegd tegen de laagste kost. De tweede constructieve heuristiek is een parallelle besparingsheuristiek. Hierin wordt vertrokken van een oplossing waarin elke klant in een apart distributiepatroon bezocht wordt. De distributiepatronen worden dan met elkaar gecombineerd zolang dit kostenbesparingen oplevert. Naast deze constructieve heuristieken wordt een verbeteringsheuristiek voorgesteld die een bestaande oplossing tracht te verbeteren door klanten uit de oplossing te verwijderen en op een andere plaats terug in te voegen.

Vervolgens worden deze heuristieken ingebed in twee metaheuristische benaderingen. De eerste is een multi-start benadering waarin de constructieve heuristieken, aangevuld met de verbeteringsheuristiek, een aantal maal herhaald worden om zo verschillende oplossingen te genereren. De beste van deze oplossingen wordt dan bijgehouden. De tweede metaheuristiek is een kolommengenererende procedure. Hierin worden distributiepatronen uit verschillende iteraties bijgehouden en vanuit deze set wordt m.b.v. mathematische programmering een subset geselecteerd die samen alle klanten bezoeken tegen minimale kosten.

Deze (meta)heuristieken lossen de eerste twee van de vijf deelproblemen op, namelijk het toewijzen van klanten aan voertuigen en aan ritten. Tijdens het uitvoeren van deze (meta)heuristieken moeten de drie resterende deelproblemen mee opgelost worden. Hiervoor gaan we als volgt tewerk. Het bepalen van de optimale volgorde van klanten in een rit gebeurt door een invoegheuristiek die klanten één vooréén invoegt in een rit, telkens op de goedkoopst mogelijke manier. De iteratiefrequenties van de verschillende ritten van een voertuig worden bepaald in een iteratieve procedure die een beperkte set mogelijke frequentiecombinaties evalueert. De tijdsschema's van de voertuigen, tenslotte, worden gegenereerd door een invoegheuristiek, waarin de verschillende ritten één voor één aan het tijdsschema worden toegevoegd.

Resultaten

De voorgestelde oplossingsmethodes worden op twee manieren geëvalueerd. Eerst wordt in een *design of experiments* een uitgebreide verzameling van probleemgevallen met uiteenlopende karakteristieken opgelost. Daarna wordt de oplossingsmethode vergeleken met twee bestaande methoden uit de literatuur. De eerste methode is een invoegheuristiek waarin de toewijzing van ritten aan voertuigen (en dus ook de vaste voertuigkost) genegeerd wordt. In vergelijking met deze methode halen wij resultaten die gemiddeld 7% goedkoper zijn voor de probleemgevallen uit onze design of experiments. De tweede methode uit de literatuur vindt de optimale oplossingen voor een aantal zeer kleine gevallen. Onze methode vindt oplossingen die gemiddeld zo'n 2% duurder zijn dan het optimum. Als we echter een onrealistische beperking op de cyclustijden uit de voorgestelde methode vervangen door de realistische veronderstelling van onze methode, vinden we zelfs oplossingen die goedkoper zijn dan de zogenaamde optima.

Daarnaast wordt ook de goede performantie vastgesteld van de invoegheuristiek die de tijdsschema's genereert, door de resultaten die deze heuristiek geeft voor 100 random gegenereerde testgevallen te vergelijken met de optimale oplossingen gegeven door een branch-and-bound algoritme.

Praktische toepassing

Naast de theoretische gevallen wordt onze oplossingsmethode ook aan de praktijk getoetst. In een project met een verdeler van papierwaren wordt een nieuwe distributiestrategie uitgewerkt met cyclische bevoorradingsschema's op basis van onze oplossingsmethode. Uit deze praktische toepassing blijkt dat onze methode, die opgezet is vanuit de veronderstelling dat de parameters zoals vraagpatronen en reistijden deterministisch en constant zijn, toch vrij robuust is m.b.t. de onzekerheden en schommelingen die deze parameters in de praktijk vertonen. Het bestuderen van deze robuustheid geeft meteen ook het belangrijkste traject voor vervolgonderzoek aan.

Summary

Problem description

The basic idea of 'Supply Chain Management' is to align the different stages of a supply chain. By integrating the decisions in planning the different activities, an improved global performance can be achieved. An example of integrating decisions can be found in 'Vendor Managed Inventory' (VMI), in which a distributor cooperates with the customers it replenishes. In a VMI-strategy, the distributor takes the responsibility for managing its customers' stocks, such that the customers no longer have to place orders. Thus, the distributor can choose how often, when and in what quantities the different customers are replenished. This freedom gives the opportunity to design more efficient vehicle routes. The optimisation problem that the distributor is faced with, namely simultaneously deciding on the replenishment quantities and designing vehicle routes to visit all customers, is known as the 'Inventory Routing' problem (IRP).

In this dissertation, the IRP is studied for the case of constant consumption of the customers. In the literature, a number of methods are suggested for this IRP with constant demand rates. These methods all consider an infinite time horizon and propose cyclic solutions. However, these methods found in the literature are not complete. The main issue is that it is assumed that enough vehicles are available, such that the problem of assigning tours to vehicles is simply ignored. A trade-off between transportation costs and holding costs is envisaged, without taking fixed vehicle costs into account.

This dissertation presents a novel cyclic modelling and solution approach for the Inventory Routing Problem with deterministic, constant customer demand rates. A fixed vehicle cost is taken into account and the fact that a single vehicle can make multiple tours, is treated explicitly. The problem of assigning tours to vehicles is thus also solved. As such, the proposed solutions do not make a two-way trade-off between transportation and holding costs, but a three-way trade-off between transportation, holding and fixed vehicle costs.

Modelling approach

To explicitly take into account the assignment of tours to vehicles, a new routing concept is presented, the so-called *distribution pattern*. This is a cyclic plan for a single vehicle that can consist of different tours that are possibly iterated with different frequencies.

Solution approach

The cyclic inventory routing problem can be seen as a combination of interdependent subproblems. A possible top-down decomposition of the problem is as follows.

1. Partition the customers over one or more vehicles.
2. Further partition the customers of a vehicle over one or more tours.
3. Determine the order in which the customers are visited within each of the tours.
4. For each vehicle, determine the iteration frequencies of the different tours it has to make.
5. Construct a delivery schedule for each vehicle to check if the distribution pattern that it should make is feasible, and determine the cost rate.

Several solution methods are presented in this dissertation that try to find good trade-offs between the different cost components, thus obtaining global cost minimizing solutions. First, two constructive heuristics are presented that build feasible solutions. The first is a sequential insertion heuristic in which customers are inserted into a solution one by one at minimal cost. The second constructive heuristic is a parallel savings heuristic. This method starts from a solution that has a separate distribution pattern for each customer. The distribution patterns are then combined as long as this results in a saving. Apart from these constructive heuristics, an improvement heuristic is also presented that tries to improve an existing solution by removing customers from the solution and then re-inserting them in another position.

Next, these heuristics are embedded into two metaheuristic frameworks. The first is a multi-start approach in which the heuristics are repeated a number of times in order to generate different solutions. The cheapest of these solutions is kept. The second metaheuristic is a column generation procedure. In this procedure, different solutions are also generated by reusing the same heuristics, but now distribution patterns from the different solutions are all kept. Information from a mathematical program is used to build the solutions in the different iterations. In the end, a mathematical program selects a subset of distribution patterns that together cover all customers at minimal costs. These (meta)heuristics deal with the two first of the five subproblems, namely partitioning customers over vehicles and over tours. In the course of these (meta)heuristics, the other three subproblems are treated as follows. Determining the optimal visiting order of customers in a tour is done by a cheapest insertion heuristic. Determining tour frequencies is done by an iterative procedure that evaluates a limited set of possible tour frequency combinations. The delivery schedules, finally, are generated by an insertion heuristic that inserts the different tours in the schedule one by one.

Computational results

The proposed solution methods are evaluated in two ways. First, an extensive set of problem instances with varying characteristics is solved in a *design of experiments*. Then, the solution approach is compared to two existing methods from the literature. The first is an insertion heuristic in which the assignment of tours to vehicles (and thus also the fixed vehicle cost) is neglected. Compared to this method, we obtain results that are on average 7% cheaper for the problem instances from our design of experiments. The second method from the literature finds the optimal solutions for a set of very small problem instances. With our solution approach, we obtain solutions that are on average 2% above the optimum. However, if we replace an unrealistic constraint on cycle times from this method by a realistic constraint from our solution approach, solutions cheaper than these 'optima' are obtained.

Furthermore, the good performance of the insertion heuristic that generates the delivery schedules is assessed. This is done by comparing the results for a set of 100 randomly generated test instances to the optimal solutions given by a branch-and-bound algorithm.

Real-life application

Apart from the theoretical problem instances, our solution approach is also applied to a practical case. In a project with a paper goods distributor, a new distribution strategy is developed in which our solution approach is used to design cyclic replenishment schemes. From this real-life application, we can conclude that, although our solution approach is developed under the assumption that parameters such as demands rates and travel times are deterministic and constant, it is still quite robust (or can easily be adjusted for robustness) w.r.t. the uncertainties and fluctuations that these parameters show in practice. Investigating this robustness offers the main avenue for further research.

Chapter 1

The cyclic inventory routing problem

1.1 Introduction

Over the last decades, there has been an increased interest in integrating management of material and information flows, both within and between companies. This has led to the emergence of concepts such as 'Enterprise Resource Planning', 'Supply Chain Management', 'Supply Chain Planning', etc. The main idea in these concepts is the collaboration between different stages in the supply chain. Decisions for the different stages are integrated and trade-offs examined, to obtain a better overall performance.

One example of supply chain collaboration can be encountered in distribution planning. Decisions at the distributor stage on routing vehicles for customer replenishment can be integrated with inventory decisions at the customer stage. Traditionally, customers manage their inventories themselves and generate replenishment orders based on their current stock level, the expected consumption in the near future and the expected delivery lead time. For the distributor, this means that replenishment orders arrive very irregularly. Since the distributor does not know exactly when his customers will be placing their orders, he is faced with the challenging task of (re)designing cost efficient delivery schedules on a daily basis. Furthermore, this uncertain situation for the distributor leads to uncertainty on the lead time, such that customers have to maintain a certain level of safety stock to avoid stocking out in the time interval between placing their order and receiving their replenishment.

Implementing the basic idea of supply chain management to these two stages leads to a situation in which much uncertainty is taken away. Customers make their inventory levels available to the distributor (usually through EDI), such that the distributor knows beforehand when his customers need a replenishment. This means that the distributor now has more freedom to select which customers will be visited when, such that more efficient vehicle routes can be designed. Because the distributor knows the customer inventory levels and makes the decisions on the timing of replenishments, less safety stock needs to be maintained at the customers.

Thus, decisions on when replenishments will occur and decisions on vehicle routes are now integrated. This leads to an overall performance that is much better for both the distributor and the customers. The distributor can design more efficient vehicle routes, while the customers' inventory is being managed with less safety stocks. This type of supply chain collaboration, integrating decisions of the distributor and customer stage, is called 'Vendor Managed Inventory' (VMI) [31] or 'Supplier Managed Inventory' (SMI).

Note that the 'distributor' and the 'customers' in the VMI concept can be found in different stages of the supply chain, sometimes even within the same company. E.g. a production facility can be seen as a distributor that has a number of regional distribution centers as its customers. The VMI concept can even be reversed to a situation where one customer receives replenishments from different distributors, e.g. a production facility that needs different parts from different suppliers. The VMI concept is thus applicable throughout the whole supply chain.

1.2 Cyclic inventory routing

In this dissertation, models and algorithms are designed for a challenging optimization problem that arises when integrating distribution and inventory management under a VMI-strategy. This problem is described here.

We consider a distribution system consisting of a distributor that has a single depot, denoted Δ, and a set of customers, denoted by S and indexed by j. The customers consume a certain product at a customer specific rate, the so-called demand rate $d_j, j \in S$. Demand rates are expressed in units per period, e.g. liter per hour, ton per month, pallets per week, ... Each customer has some storage space available for stocking the product, with a capacity of κ_j units, $j \in S$. The customers need to be repeatedly replenished from the depot without ever stocking out.

Since we are assuming a VMI-policy, the distributor has the responsibility of deciding when to replenish which customers, and in what quantities. Thus, decisions of both distribution and inventory management are integrated. Since the distribution part of this challenging integrated decision consists of developing routes along which the vehicles have to travel, this problem is called the 'Inventory Routing Problem' (IRP) in literature. The IRP appears in literature in a number of variations, depending on the problem characteristics. Section 1.3 below gives an overview of these different versions and the existing solution approaches.

The main assumption in our work is that the customer demand rates d_j are constant over time, i.e. if a customer consumes 20 units this week, he will also consume 20 units next week, and he will still consume 20 units every week in six months time. This assumption of constant demand rates has a very important consequence on the solution approach. Suppose an efficient vehicle route is found that replenishes a subset of customers S_r, such that the quantities q_j delivered cover demand at these customers for the same amount of time T, i.e. $q_j = T \cdot d_j, \forall j \in S_r$. Because the customer demand rates do not change, this vehicle route will still be as efficient the next time these customers need to be replenished, i.e. T periods later. Thus, the same vehicle route and delivery quantities remain valid and can repeatedly be used. In other words, for constant demand rates, a cyclic solution approach is appropriate. Cyclic solution approaches offer cost efficient solutions which are stable and predictable over time. This predictability reduces operational complexity and variability both for the distributor (vehicle route design) and the customers (delivery handling).

To make sure that all customers in a tour receive a quantity that covers their demand for the same period of time, we assume that the fair-share rule is being applied to determine delivery quantities. This means that the vehicle load will be divided over the customers in a tour proportional to their demand rates. For example, if a customer with a demand rate of 30 units per day is replenished in a single tour together with another customer consuming 10 units per day, then $30/(30+10) = 75\%$ of the vehicle load is always delivered to the first customer, and $10/(30+10) = 25\%$ of the vehicle load is always delivered to the second customer, regardless of the actual vehicle load.

Another important assumption that we are making is that designing the fleet of vehicles is part of the problem, or in other words, the number of vehicles to be used in the distribution system has to be endogenously determined. We assume a homogenous fleet of vehicles with a capacity of κ units.

We consider a very general cost structure, consisting of five components. Since a cyclic approach is used, solution quality is expressed in terms of *cost rates*, i.e. costs per period.

1. There is a fixed cost per vehicle that is being used. This fixed cost is ψ euro per period. It is accounted regardless of the activity of the vehicle. For a vehicle that has to wait in the depot in between two tours, a cost of ψ euro per period is still incurred. This cost component reflects the opportunity cost and/or the time cost of the vehicle[1].

2. The second cost component is the variable transportation cost. In the routing literature, two approaches are often considered. Either (i) the transportation cost and time between every pair of locations are given, or (ii) the distance between any pair of locations is given, and a constant

[1] The time cost of a vehicle equals the sum of all fixed costs (fixed depreciation cost, wages, insurance, etc.) divided by the expected annual number of working hours [15].

transportation cost of δ euro per kilometer and a constant speed of ν kilometer per hour are assumed. Our solution approach supports both approaches.

3. Every time a vehicle leaves the depot to make a replenishment tour, a fixed dispatching cost of φ_Δ euro is accounted. This is the third cost component. It reflects the costs that are made for loading a vehicle.

4. The fourth cost component is the cost of delivery incurred at the customer. A fixed cost of φ_j euro is accounted for each delivery made to customer $j \in S$, reflecting the costs made for unloading the vehicle and handling the delivered goods.

5. The fifth and last cost component is the stock holding cost at the customers. For every customer $j \in S$, a holding cost of η_j euro per unit of product per period is accounted.

The resulting problem that needs to be solved, is a Cyclic Inventory Routing Problem (CIRP). A cyclic distribution scheme has to be designed such that the total cost rate, consisting of all five components, is minimized.

The final important assumption that we are making is that replenishment of a customer is always done by the same vehicle, in the same tour. However, this does not mean that a vehicle can only make a single tour. A vehicle can make a whole set of different tours as long as driving and working time restrictions are not violated. In fact, a single vehicle can even make different tours with different frequencies. E.g. to efficiently use vehicle capacity, a tour to a customer with a high demand rate that is located close to the depot may be done every other day, while the same vehicle may replenish a far-away customer with a lower demand rate only once every week. Such a cyclic distribution scheme for a single vehicle consisting of multiple tours, repeated with different frequencies, will be called a 'distribution pattern'. This concept is discussed in detail in Chapter 2.

Possible extensions

In this dissertation, a modelling and solution approach are presented for the above 'basic' version of the cyclic inventory problem. However, some extensions can easily be made to adapt the approach to problem variants.

The basic version considers a single depot, which covers one-to-one, one-to-many and many-to-one VMI relationships. To cover many-to-many VMI relationships, the approach needs to be extended to multiple depots.

The approach presented in this dissertation can deal with multiple products, as long as demand rates can be expressed in a common unit and the different products can be transported together, i.e. in the same compartment of a vehicle. Dealing with different products that cannot be transported together (e.g. different crude oil derivates) requires the approach to be extended with some extra transportation constraints.

The presented solution approach works with a fleet of homogeneous vehicles. The extension to multiple vehicle types and heterogeneous fleets can easily be made.

The main assumption for using the cyclic approach is that customer demand rates are constant. Of course, this assumption is not always valid. However, we will present a real-life case study, in which we will indicate that our cyclic solution approach is inherently robust. Investigating how the cyclic approach can be extended to take demand uncertainty into account a priori is an interesting avenue for future research. A preliminary study on this issue has already been undertaken and is reported in [26].

1.3 Literature review

The problem of integrating inventory and distribution decisions has been approached in different ways depending on inventory policies used at the customers, service level restrictions, and the time horizon considered. This section is devoted to a brief review of papers which are representative of models and solution approaches proposed for the Inventory Routing Problem (IRP).

Bell *et al.* (1983) [8] are among the first to integrate inventory management with vehicle scheduling. They use mathematical optimization to solve the daily vehicle scheduling problem with an objective of minimizing vehicle mileage while avoiding stock-outs. Federgruen and Zipkin (1984) [17] study the combined inventory allocation and vehicle routing problem. They consider a myopic single-period problem with random demands and a fixed fleet size. They calculate inventory costs as a balance between holding costs and shortage costs. Two approaches are described to solve the problem. In the first approach, the problem is separated into the inventory allocation problem and a number of TSP's, and then solved with an iterative interchange heuristic. The second approach is an exact algorithm based on Benders' decomposition. Dror and Ball (1985, 1987) [14, 13] consider the long-term IRP and reduce it to a single-period problem by defining single-period costs that reflect long-term costs through incorporating stock-out probabilities and anticipated stock-out costs. They propose and compare two solution algorithms for the resulting short-period problem.

The seminal papers mentioned above have incited a whole body of literature on inventory routing, most of them considering stochastic demands and short time horizons. For a recent survey, see Kleywegt *et al.* (2002) [22]. However, if customer demand can be assumed deterministic and constant, then cyclic solution approaches over an infinite planning horizon are more appropriate. The advantage of cyclic approaches is the stability and predictability over time, and the reduced variability and operational complexity for the distributor and its customers. Papers using cyclic solution approaches are discussed below. Table 1.1 offers a summary of their main features.

Larson [23] is the first to explore a cyclic approach for the long-term IRP. He considers the so-called Strategic Inventory Routing Problem (SIRP) in which the objective is to minimize the required fleet size. For this, he uses a savings-based heuristic that assigns customers to clusters. For replenishment, it is assumed that all customers in a cluster are visited in a single route. For the actual operational routing of the selected fleet, he refers to existing stochastic inventory routing models because the customer demands are stochastic in the short term.

Webb and Larson (1995) [32] generalize Larson's savings heuristic for the strategic IRP. Instead of having all customers in a cluster visited in a single route, they introduce the concept of *routesets* and the period and phase of customer replenishment. For any cluster of customers, a routeset consists of a number of component routes arranged in a specific order. Each route visits a subset of the customers in the cluster. The period of a customer is then the number of routes between consecutive replenishments, while the phase of a customer is the number of routes in the routeset before the first route that replenishes this customer. By using this more general routing concept, the average vehicle requirement can be significantly reduced.

Anily and Federgruen (1990) [5] do not consider vehicle fleet costs, but instead aim at minimizing the long-run average transportation and inventory costs. They assume that customer demand rates d_j are multiples of a common quantity μ, i.e. $d_j = k_j\mu$, and define a demand point as a point in the plane facing a demand rate of μ. Customer j thus consists of k_j demand points. In their solution approach, they restrict themselves to the class of replenishment strategies in which demand points are partitioned into regions. Because a customer consists of demand points, it may thus appear in more than one region. Each time a demand point in a given region receives a delivery, all other demand points in the region are visited as well, by the same vehicle. This partitioning method is used to derive lower and upper bounds on the long-run average total cost. They show that these bounds are asymptotically tight for this regional partitioning strategy when the number of demand points tends to infinity. This work is extended in their 1993 paper [6] to a two-echelon distribution problem where the central warehouse keeps system stocks instead of being a mere transhipment point. Anily (1994) [2] develops a lower bound and a heuristic for the more general case in which holding cost rates are customer-specific.

Using ideas similar to those of Anily and Federgruen, Gallego and Simchi-Levi (1990) [19] evaluate the long run effectiveness of direct shipping (i.e. having a separate route for each customer). They conclude that direct shipping is at least 94% effective over all inventory routing strategies whenever the minimal economic lot size is at least 71% of truck capacity. This shows that direct shipping becomes a bad policy when many customers require significantly less than a truck load, making more complicated routing policies the appropriate choice for most real-life applications.

Bramel and Simchi-Levi (1995) [9] also study fixed partition policies for the deterministic inventory routing problem with an unlimited number of vehicles. They propose a location based heuristic to choose a fixed partition, based on the capacitated concentrator location problem (CCLP). The tour through each subset of customers is constructed while solving the CCLP, using a nearest insertion heuristic.

Chan, Federgruen, and Simchi-Levi (1998) [11] also analyze fixed partition policies. They derive asymptotic worst-case bounds on the performance of these policies. They also propose a heuristic based on the CCLP, similar to that of Bramel and Simchi-Levi [9], for determining a fixed partition of the set of customers.

Anily and Bramel (2004) [3, 4] derive a deterministic lower bound on the cost of the optimal fixed partition policy and present probabilistic analyses of the performance of this bound.

Gaur and Fisher (2004) [20] present a real-life case of a deterministic inventory routing problem with time varying demand. They propose a randomized heuristic to find a fixed partition policy with periodic deliveries. For the supermarket chain under consideration, distribution cost savings up to 20% are expected.

Viswanathan and Mathur (1997) [30] adopt a stationary nested joint replenishment policy for the inventory-routing problem with multiple products and deterministic demand rates. A policy is called *stationary* if replenishments are made at equally spaced points in time. A *nested* policy means that if the replenishment interval of a given customer is larger than that of another customer served by the same vehicle, the former is a multiple of the latter. They present an insertion heuristic with a powers-of-two policy for the replenishment intervals.

Qu *et al.* (1999) [25] consider a central warehouse and several suppliers. The warehouse replenishes its stock by dispatching vehicles to collect goods from the suppliers. The total cost consists of transportation costs (for dispatching, stopover and routing) and inventory costs (for ordering, holding and backlog). Their work is restricted to a modified version of the periodic review policy, in which the replenishment period for all items is an integer multiple of a base period T. The solution approach is to decompose the problem into two parts, an inventory problem and a transportation problem. The overall solution is found by iterating between these two problems. A general lower bound to the model is constructed, and a better lower bound for the special case when each supplier produces a unique item.

Sindhuchao *et al.* (2005) [28] consider a system in which a set of items is collected from a set of suppliers by a fleet of vehicles facing a frequency constraint. They adopt a fixed partition policy to minimize the long-run average inventory and transportation costs. They present (i) a column generation approach that gives a lower bound, (ii) a branch-and-price algorithm that gives the optimal partition for (very) small problem instances, and (iii) some heuristics that give near-optimal solutions for medium sized problems (up to 50 customers).

In Table 1.1, the main features of the papers found in the literature on inventory routing with an infinite time horizon are summarized. In these papers, the routing concept that is mostly used is the single route, i.e. it is assumed that a vehicle is available for each of the routes designed. In other words, the assignment of routes to vehicles is not being considered. Viswanathan and Mathur [30] and Qu *et al.* [25] make the generalization to nested routes, in which customers in a route are not necessarily visited at every iteration of the route. The most general routing concept is found in the routesets of Webb and Larson [32]. When using routesets, the assignment of routes to vehicles is explicitly considered. However, the concept of routesets is only used for determining the required fleet size and not for the actual routing of the vehicles. In the approach presented in this dissertation, fixed vehicle costs are considered and thus the assignment of routes to vehicles has to be taken into account. In the generalized routing concept that we use, the so-called distribution pattern, a vehicle is allowed to make multiple tours at different iteration frequencies.

The limited customer storage capacity (CCAP) restriction is ignored in most solution approaches. This means that a customer is assumed to have enough capacity to store any delivery quantity that the distributor may decide to bring. This is not true in most real-life cases and therefore, in our approach, both the vehicle capacity and the customer storage capacities are used to determine maximal delivery quantities.

Because stock-outs have to be avoided, the maximal delivery quantities result in a restriction on the time between consecutive replenishments of a customer, i.e. in a maximal cycle time. On the other hand, there is also a minimal cycle time, i.e. a minimal amount of time between consecutive deliveries because the vehicle needs some time to drive to other customers and to the depot before it can come back. Apart from the paper of Sindhuchao *et al.* [28], this minimal cycle time is not taken into account. It is thus assumed that the time needed for loading, unloading and driving around in a route is always negligible in comparison to the actual cycle time of that route. In the paper of Sindhuchao *et al.* [28], a minimal cycle time is imposed through a vehicle frequency constraint, stating that a vehicle cannot make more than a certain number of tours per period, e.g. no more than 10 tours per week. This constraint is unrealistic because it does not depend on how long it actually takes to make the tours. As such, our approach is the first in the literature that uses a (realistic) minimal cycle time.

1.4 Contribution

The existing literature on cyclic inventory routing has ignored the possibility of one vehicle performing several routes to different customer subsets. Although

Table 1.1: Features of cyclic IRP papers in the literature

Authors	Route design	Reorder policy	CCAP	VCAP	Min. cycle time	Application	Problem size
Larson [23]	single route	up-to reordering	yes	yes	-	yes	14
Webb and Larson [32]	routesets	up-to reordering	yes	yes	-	-	100
Anily and Federgruen [5, 6]	single route	zero reordering	-	yes	-	-	10000
Anily [2]	single route	zero reordering	-	yes	-	-	1000
Gallego and Simchi-Levi [19]	direct shipping	zero reordering	-	yes	-	-	-
Bramel and Simchi-Levi [9]	single route	zero reordering	-	yes	-	-	50
Viswanathan and Mathur [30]	nested routes	zero reordering	-	yes	-	-	1000
Chan et al. [11]	single route	zero reordering	-	yes	-	-	200
Qu et al. [25]	nested routes	periodic review	-	-	-	-	50
Anily and Bramel [3, 4]	single route	zero reordering	-	yes	-	-	-
Gaur and Fisher [20]	single route	zero reordering	-	yes	-	yes	207
Sindhuchao et al. [28]	single route	zero reordering	-	yes	vehicle frequency	-	50

it is not explicitly stated that each route has to be performed by a separate vehicle, the solution approaches developed so far do not consider assigning tours to vehicles. When assigning tours to vehicles, the tour frequencies have to be aligned, which could increase the total cost of the solution. In the literature, this problem has been circumvented by excluding fixed vehicle costs through the assumption that ‘enough’ vehicles are available. One exception to this is the work of Larson [23] and Webb and Larson [32]. However, they only determine the fleet size and do not determine the actual vehicle routes.

In the short run, when the vehicle fleet size is fixed, it makes sense to limit the analysis to variable costs only. However, the cyclic inventory routing problem has to be analyzed from a long-term perspective, in which the vehicle fleet is variable by definition. Therefore, decisions on the vehicle fleet size have to be incorporated and a fixed vehicle cost has to be taken into account.

This dissertation aims at offering a novel approach to cyclic inventory routing by explicitly taking fixed vehicle costs into account. This is a major departure from the existing literature, significantly increasing the relevance of cyclic inventory routing to practitioners. Moreover, the approach takes into account realistic restrictions such as limited customer storage capacities and minimal cycle times. The solution approach that will be presented is highly generic and can easily be extended to accommodate additional real-life side-constraints, such as time windows, multiple vehicle types, etc.

The remainder of this dissertation is organized as follows. Chapter 2 presents the modelling approach. The generalized routing concept of distribution patterns is introduced. This concept adds to complexity considerably, because the different tours that a vehicle makes, can have different frequencies. First, these frequencies have to be determined and then schedules have to be constructed that indicate the sequence in which the different tours should be made.

Chapter 3 describes the solution approach. It begins with the presentation of two constructive heuristics, an insertion heuristic and a savings heuristic, and an improvement heuristic. Then, the complex subproblems of determining tour frequencies and constructing delivery schedules are tackled. Finally, the proposed heuristics are embedded in two metaheuristic frameworks: a simple multi-start framework and a more advanced column generation framework.

Computational experiments of our solution approach are presented in Chapter 4. An extensive design of experiments is set up to study the impact of different characteristics such as the routing concept being used (single tours vs. distribution patterns), the vehicle capacity, etc. The solution approach is also compared to two existing approaches found in the literature to evaluate its performance.

After the theoretical problem instances, our solution is applied to a real-life case in Chapter 5. In the case study, a cyclic distribution strategy is set up for a paper goods distributor, adopting the concept of distribution patterns.

Chapter 6 finishes this dissertation with conclusions from the current work and some directions for future research.

Chapter 2

Modelling approach: Distribution patterns

The existing models for routing problems mostly use ‘tours’ as a basic construct to build solutions. In a tour, the vehicle leaves the depot, visits a set of customers one after the other, and then returns to the depot. Modelling routing problems with this construct results in solutions where each vehicle makes only one tour. While this may be appropriate for short-term routing problems, it certainly is not for the longer term. Since we consider an infinite time horizon, we drop the assumption of a single tour per vehicle and allow vehicles to make different tours. Thus, the tour concept is generalized to ‘multi-tours’. Next, we generalize the concept of multi-tours even further by introducing different frequencies for the tours made by the same vehicle. This results in the concept of so-called ‘distribution patterns’.

In most real-life situations, the driving time of the vehicles is restricted, limiting the number of feasible vehicle routes. A distinction is therefore made between situations with and without these driving time restrictions. The following sections give a formal description of the different routing concepts for both situations. In Section 2.1, the relatively easy concept of tours is discussed in the context of cyclic inventory routing. This is generalized in Section 2.2 to multi-tours, and finally, in Section 2.3, to distribution patterns. Throughout, an illustrative example is used that presents both the opportunities and complications that arise when adopting the concept of distribution patterns.

2.1 Modelling with tours

Consider a vehicle replenishing a set of customers S. In terms of travel distance, the most effective way to supply these customers is to travel along the shortest tour that visits the depot Δ and all of the customers in S, or the ‘TSP-tour’ through S plus the depot, $TSP(S+\Delta)$. Cyclically repeating this tour gives a solution for the infinite time horizon. The time between two consecutive iterations of the tour is called the 'cycle time' T.

The vehicle needs some time for loading at the depot, denoted t_Δ, for driving around in the tour to all customers, denoted $T_{TSP(S+\Delta)}$ and for unloading at all of the customers, denoted $\sum_{j\in S} t_j$. The tour cannot be restarted before it is finished, so the total time needed to complete a tour gives a lower bound on the cycle time.

Sometimes customers restrict their delivery frequency f_j. This results in another lower bound on the cycle time. Suppose a tour can be finished within one hour, but customer j in that tour wants to be replenished at most once every two hours ($f_j = 1/2$). Then the cycle time for the whole tour has to be at least two hours ($= 1/f_j$).

Thus, the minimal cycle time for replenishing the set of customers S in a single tour, denoted by T_{min}, is given by the following formula.

$$T_{min} = \max\left(\left(t_\Delta + T_{TSP(S+\Delta)} + \sum_{j\in S} t_j\right),\ \max_{j\in S}\frac{1}{f_j}\right) \tag{2.1}$$

The cycle time of a tour is not only bound from below. Due to capacity restrictions, there is also an upper bound.

The first capacity restriction is related to the limited vehicle capacity. The maximum quantity a vehicle can distribute over the customers in a tour is exactly the vehicle capacity κ. To avoid stock-outs, the vehicle has to return to its customers before they have consumed this full truckload. Knowing that the vehicle load is divided over the customers in a tour according to a fair-share mechanism, the vehicle capacity divided by the cumulative demand rate of all customers in the tour, $\sum_{j\in S} d_j$, gives the maximal time between two consecutive deliveries without customer stock-outs, and thus an upper bound on the cycle time of the tour.

Other capacity restrictions are related to the limited storage capacities of the customers. If the storage capacity κ_j of a customer is smaller than the maximal delivery quantity resulting from the vehicle capacity, this further reduces the maximal cycle time. E.g. consider two customers with demand rates of 20 and 10 units per hour, served in a single tour by a vehicle with a capacity of 120 units. The maximal cycle time of this tour is 4 hours, with a delivery of 80 units to the first customer and a delivery of 40 units to the second customer. Now suppose the first customer can only hold up to 60 units of stock. Then the maximal cycle time reduces to 3 hours, bringing 60 units to the first customer, and 30 units to the second customer. In this case, the available vehicle capacity cannot be fully used and it makes sense to add a third customer to the tour, if possible.

The following formula gives the maximal cycle time for replenishing the set of customers S in a single tour, denoted by T_{max}.

$$T_{max} = \min\left(\frac{\kappa}{\sum_{j\in S} d_j}, \min_{j\in S}\frac{\kappa_j}{d_j}\right) \tag{2.2}$$

Obviously, a tour is only feasible if its minimal cycle time is not greater than its maximal cycle time: $T_{min} \leq T_{max}$.

The cost rate of a tour, denoted C, consists of the five components that we discussed in Section 1.2.

1. The first component is the fixed operating cost of the vehicle, ψ euro per period.
2. The second cost rate component is the variable transportation cost. Per iteration, the vehicle travels along the TSP-tour through the depot and the set of customers once, at a cost of $C_{TSP(S+\Delta)}$ euro. This results in a variable transportation cost of $C_{TSP(S+\Delta)}/T$ euro per period.
3. The third cost rate component is the fixed vehicle loading and dispatching cost. The vehicle is loaded and dispatched once at a cost of φ_Δ euro per iteration of the tour, giving φ_Δ/T euro per period.
4. The fourth cost rate component is the fixed unloading cost at the customers. Per cycle, there is one delivery at each of the customers, so this cost component amounts to $\sum_{j\in S}\varphi_j$ euro per iteration of the tour, or $\sum_{j\in S}\varphi_j/T$ euro per period.
5. The fifth and last cost rate component is the stock holding cost at the customers. The quantity q_j delivered to customer $j \in S$ covers demand for a whole cycle (i.e. until the next delivery), so $q_j = d_j T$ units. The average stock level at customer j during a cycle is then $q_j/2$ units. With a storage cost of η_j euro per unit per period, the stock holding cost at customer j is: $\eta_j \cdot q_j/2$ or $\eta_j d_j \cdot T/2$ euro per period. The total stock holding cost at all customers in S is then $\frac{T}{2}\sum_{j\in S}\eta_j d_j$ euro per period.

The total cost rate of the tour replenishing the set of customers S is given in the following formula.

$$C = \psi + \frac{1}{T}\left(\varphi_\Delta + C_{TSP(S+\Delta)} + \sum_{j\in S}\varphi_j\right) + \frac{T}{2}\sum_{j\in S}\eta_j d_j \tag{2.3}$$

This tour cost rate varies with the tour cycle time. Each tour thus has a theoretical optimal cycle time, for which the cost rate is minimal. This occurs when $\frac{\partial C}{\partial T} = 0$, where holding costs at the customers (component 5), which increase when cycle time increases, are balanced with the sum of transportation costs and fixed tour costs (components 2, 3 and 4), which decrease when cycle time increases. This is a generalization of the well-known 'Economic Order Quantity' model, and therefore this optimal cycle time is called the EOQ cycle time and denoted T_{eoq}.

$$T_{eoq} = \left(\frac{\varphi_\Delta + C_{TSP(S+\Delta)} + \sum_{j \in S} \varphi_j}{\frac{1}{2} \sum_{j \in S} \eta_j d_j} \right)^{1/2} \tag{2.4}$$

Unfortunately, this EOQ cycle time is not always feasible. In many cases, T_{eoq} will be outside the interval $[T_{min}, T_{max}]$ of feasible cycle times. When this is the case, the best feasible cycle time T^* is the feasible cycle time closest to T_{eoq}, which is exactly the maximal or minimal cycle time.

$$T^* = \begin{cases} T_{min} & \text{if } T_{eoq} < T_{min}, \\ T_{eoq} & \text{if } T_{min} \le T_{eoq} \le T_{max}, \\ T_{max} & \text{if } T_{max} < T_{eoq}. \end{cases} \tag{2.5}$$

Illustrative example

Throughout this chapter, a 4-customer example is discussed to illustrate the opportunities and complexities of using the different routing concepts for cyclic inventory routing. Distances and demand rates of the example are shown in Figure 2.1. A vehicle with a capacity of 120 units is available for product replenishment from the depot. For simplicity, it is assumed here that (i) the loading and unloading times of the vehicle are negligible ($t_\Delta = t_j = 0$), (ii) customers have no storage capacity restrictions ($\kappa_j = \kappa$), and (iii) customers do not impose delivery frequencies ($f_j = +\infty$).

The cost parameters are as follows:

1. The fixed vehicle cost: $\psi = 20$ euro per hour.

2. For the variable transportation cost, an average speed of 50 km per hour and a cost of 1 euro per kilometer are assumed.

3. No vehicle loading and dispatching cost is accounted: $\varphi_\Delta = 0$.

4. No vehicle unloading cost is accounted: $\varphi_j = 0, j = 1..4$.

5. All four customers have the same storage cost of 0.15 euro per unit per hour.

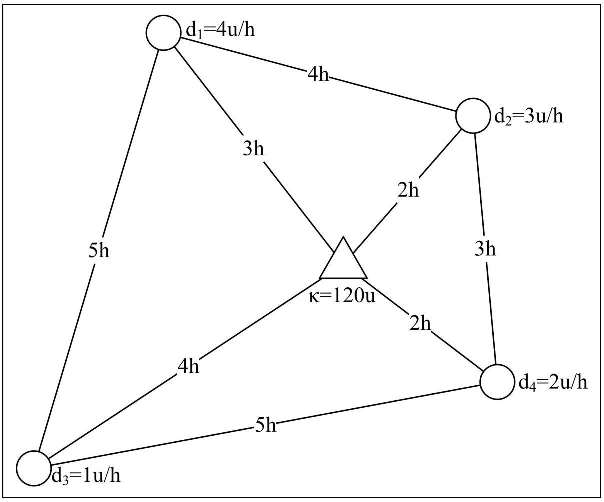


Figure 2.1: Illustrative example with 4 customers

The shortest tour through the depot and all four customers goes from the depot to customer 2, on to customers 1, 3 and 4 next and then back to the depot. The minimal cycle time of this tour is its travel time, i.e. 18 hours. The maximal cycle time due to the limited vehicle capacity is $120/(4+3+2+1) = 12$ hours. Because the minimal cycle time exceeds the maximal cycle time, this simple tour solution is infeasible. Therefore, when using the single tour concept for routing vehicles, a second vehicle would be necessary for replenishing the four customers in this example. Alternatively, a bigger vehicle can be considered. Note that the suggested tour becomes feasible with a vehicle capacity κ of 180 units. However, when generalizing the routing concept in the following sections, we will show that no second vehicle or larger vehicle is needed at all.

2.1.1 Tours under driving time restrictions

So far, we have assumed that a vehicle can make a tour at any time of day. This assumption may be valid for some industrial applications, such as the distribution of crude oil derivates or liquefied gases, raw material replenishment, etc. However, in most real-life situations, deliveries can only occur during the day. In Chapter 5, a real-life application of our cyclic inventory routing tool is presented. In that case, the products are consumed day and night every day, while replenishments are allowed only during the day, on weekdays, so a vehicle can only drive 8 hours a day, and 5 days a week.

When driving time restrictions of 8 hours per day apply, we assume that the cycle time of a tour has to be an integer number of days and thus that a tour cannot be made more than once per day. Similarly, when driving is only permitted on weekdays, we assume that the tour cycle time is an integer number of weeks and thus that a tour is made at most once per week. Cycle times therefore have to be 'translated' from hours to a number of days.

The minimal cycle time under driving time restrictions is denoted by T_{MIN}. It is 1 day if completing the tour takes less than 8 hours. A minimal cycle time of more than one day is not permitted, because we assume that the vehicle drivers have to be back in the depot at the end of their 8-hour working day.

$$T_{MIN} = \begin{cases} 1 & \text{if } T_{min} \leq 8, \\ +\infty & \text{otherwise.} \end{cases} \tag{2.6}$$

If driving time restrictions are imposed, we assume that customer demand rates are expressed in units per day (or week) instead of units per hour. The maximal cycle time under driving time restrictions, denoted T_{MAX}, is then an integer number of days (or weeks) given by a straightforward extension of the original formula for the maximal cycle time T_{max}. Obviously, a maximal cycle time of zero days (or weeks) indicates that a tour is infeasible.

$$T_{MAX} = \min\left(\left\lfloor \frac{\kappa}{\sum_{j\in S} d_j} \right\rfloor, \min_{j\in S} \left\lfloor \frac{\kappa_j}{d_j} \right\rfloor\right) \tag{2.7}$$

The optimal cycle time T_{EOQ} is given by rounding T_{eoq} to the closest integer number of days (or weeks).

Illustrative example

In the illustrative example, the driving time restrictions that we impose are the 8-hours driving per day constraints. Further, it is assumed that the customers in the example are retail outlets such that demand only occurs during these 8 hours per day.

The single tour solution suggested above takes 18 hours. This violates the 8-hour driving constraint, such that this tour is always infeasible, even if the vehicle capacity would be 180 units or more.

2.2 Modelling with multi-tours

As discussed in the literature review of Section 1.3, the concept of simple tours is adequate for vehicle routing with a short time horizon, since vehicles usually cannot make more than one tour within the short time horizon considered. Fleischmann [18] was the first to break this assumption and allow vehicles to make more than one tour in a vehicle routing problem.

The cyclic inventory routing problem that we are studying has an infinite time horizon, so it is certainly a good idea to take the opportunity of using the same vehicle for more than one tour. The 'multi-tour' concept in this context is discussed here.

Consider a vehicle making a set of n tours, $T_1, \ldots, T_n$, visiting customer subsets $S_1, \ldots, S_n$. Since we do not consider split delivery, these customer subsets are disjoint. The vehicle repeatedly makes these tours one after the other. After finishing the last tour, T_n, the cycle is finished, and a new cycle starts with the next iteration of tour T_1.

The vehicle needs time for loading at the depot n times, making all n tours and unloading at all of the customers. Further, the customers may impose maximal delivery frequencies f_j. This leads to a minimal cycle time T_{min} for a multi-tour, given by the following formula.

$$T_{min} = \max\left(\sum_{i=1}^{n}\left(t_\Delta + T_{TSP(S_i+\Delta)} + \sum_{j\in S_i} t_j\right),\ \max_{i=1..n}\max_{j\in S_i}\frac{1}{f_j}\right) \tag{2.8}$$

When making tour T_i, the vehicle can deliver at most a full truckload κ to the customers in this tour. The vehicle has to return to these customers before they run out of stock. In the meantime however, the vehicle has to make the other tours as well. The cycle time of the whole multi-tour is therefore at most $\kappa / \sum_{j\in S_i} d_j$.

The limited storage capacities of the visited customers also restrict the maximal cycle time T_{max} of the multi-tour, which is then as follows.

$$T_{max} = \min\left(\min_{i=1..n}\frac{\kappa}{\sum_{j\in S_i} d_j},\ \min_{i=1..n}\min_{j\in S_i}\frac{\kappa_j}{d_j}\right) \tag{2.9}$$

To ensure feasibility, the maximal cycle time cannot be smaller than the minimal cycle time: $T_{min} \leq T_{max}$.

The formula for the cost rate of a multi-tour C, consisting of the five components, is obtained by a straightforward extension of the formula for the simple tour cost rate that is derived in the previous section.

$$C = \psi + \frac{1}{T}\sum_{i=1}^{n}\left(\varphi_\Delta + C_{TSP(S_i+\Delta)} + \sum_{j\in S_i}\varphi_j\right) + \frac{T}{2}\sum_{i=1}^{n}\sum_{j\in S_i}\eta_j d_j \tag{2.10}$$

Multi-tours have a theoretical optimal cycle time, T_{eoq}, for which this cost rate is minimal, namely when holding costs are balanced with the sum of transportation costs and fixed tour costs.

$$T_{eoq} = \left(\frac{\sum_{i=1}^{n} \left(\varphi_\Delta + C_{TSP(S_i+\Delta)} + \sum_{j \in S_i} \varphi_j \right)}{\frac{1}{2} \sum_{i=1}^{n} \sum_{j \in S_i} \eta_j d_j} \right)^{1/2} \tag{2.11}$$

As for single tours, this EOQ cycle time may be infeasible. If this is the case, the best feasible cycle time T^* of the multi-tour is the one closest to T_{eoq}, i.e. the maximal or minimal cycle time.

Illustrative example

For the illustrative 4-customer example, we consider the multi-tour solution in which the vehicle makes two separate tours. The first tour visits customers 1 and 3, the second tour visits customers 2 and 4. This way, the total demand rate is balanced over the different tours. The travel time of this solution is 19 hours (12 hours for the first tour + 7 for the second). The maximal cycle time of the first tour is $120/(4+1) = 24$ hours, the maximal cycle time of the second tour is $120/(3+2) = 24$ hours. The maximal cycle time of the multi-tour is the minimum of both, i.e. 24 hours. This maximal cycle time is larger than the minimal cycle time, so the solution is feasible.

With the cost parameters given above, the optimal cycle time of this multi-tour is: $T_{eoq} = \sqrt{\frac{50 \cdot 12 + 50 \cdot 7}{0.15 \cdot 5}} = 35.6$ hours. This is way above the maximal cycle time of 24 hours, so the best feasible cycle time is exactly 24 hours. The cost rate of the multi-tour is then: $C = \psi + \frac{50 \cdot 19}{T} + T \cdot 0.15 \cdot 5 = 20.00 + 39.58 + 18.00 = 77.58$ euro per hour.

One iteration of the multi-tour cycle is depicted in Figure 2.2. During one cycle of 24 hours, all customers receive one delivery, in a quantity that covers exactly 24 hours of demand. At time $t = 0$, the vehicle leaves the depot fully loaded to make the first tour. After 3 hours, the vehicle arrives at customer 1 and delivers 96 units, and after 8 hours, the vehicle delivers the remaining 24 units to customer 3. Four hours later, the vehicle is back in the depot. It is then fully reloaded and it immediately leaves for the second tour. At time $t = 14h$, a delivery of 72 units is made at customer 2, and 3 hours later, the remaining 48 units are delivered to customer 4. After 19 hours, the vehicle is back in the depot. It then has 5 hours left before re-initiating the same sequence.

Table 2.1 gives an overview of the main characteristics of this multi-tour solution.

Alternative solution

In another possible multi-tour solution for our illustrative 4-customer example, the vehicle now makes three separate tours. The first tour visits customer 1,

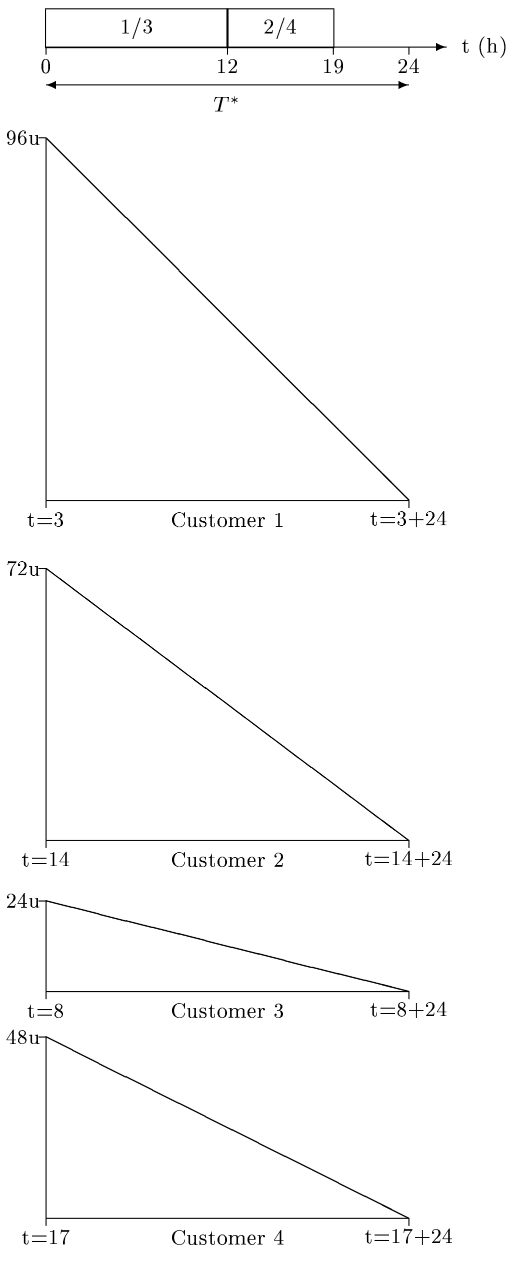


Figure 2.2: Schedule and stock levels for the multi-tour solution

Table 2.1: Characteristics of the multi-tour solution

T_{min}	19h
T_{max}	24h
T_{eoq}	35.6h
Cycle time	24h
q_1	96 units
q_2	72 units
q_3	24 units
q_4	48 units
Transport cost rate	39.58 €/h
Holding cost rate	18.00 €/h
Cost rate	77.58 €/h
Idle time	5/24 = 20.8%
Vehicle occupation	79.2%
Capacity utilization	100%

the second tour visits customers 2 and the third tour visits customers 3 and 4. The travel time of this solution is 21 hours (6 hours for the first tour + 4 for the second + 11 for the third). The maximal cycle time of this multi-tour is $\min(120/4, 120/3, 120/(2+1)) = 30$ hours. The maximal cycle time is larger than the minimal cycle time, so this solution is feasible as well.

The optimal cycle time of this multi-tour is: $T_{eoq} = \sqrt{\frac{50\cdot 21}{0.15\cdot 5}} = 37.4$ hours. This is above the maximal cycle time of 30 hours, so the best feasible cycle time is 30 hours. The cost rate of this multi-tour is then: $C = \psi + \frac{50\cdot 21}{T} + T\cdot 0.15\cdot 5 = 20.00 + 35.00 + 22.50 = 77.50$ euro per hour. Although the vehicle travels more per cycle, this second multi-tour solution turns out to be slightly cheaper. This is because a longer cycle time, closer to the optimal cycle time, and thus a better cost trade-off can be obtained. Table 2.2 shows the main characteristics of this alternative multi-tour solution.

Mixed integer formulation

This paragraph presents a mixed integer formulation for the cyclic inventory routing problem with multi-tours. For this formulation, the following notations are introduced.

- V is a set of vehicles, indexed by v.
- S^+ is the set of 'locations', indexed by j, k and l. It consists of the set of customers S and the depot Δ.
- c_{jk} is the variable transportation cost between locations j and k.
- t_{jk} is the travel time between locations j and k.

Table 2.2: Characteristics of the alternative multi-tour solution

T_{min}	21h
T_{max}	30h
T_{eoq}	37.4h
Cycle time	30h
q_1	120 units
q_2	90 units
q_3	30 units
q_4	60 units
Transport cost rate	35.00 €/h
Holding cost rate	22.50 €/h
Cost rate	77.50 €/h
Idle time	9/30 = 30.0%
Vehicle occupation	70.0%
Capacity utilization	83.3%

In the mixed integer formulation, shown in Table 2.3, the following variables are used.

- Y_v is a binary variable that indicates whether vehicle $v \in V$ is being used or not. This variable is needed to determine the vehicle fleet size.
- T_v is the cycle time of the multi-tour that vehicle $v \in V$ makes.
- X_{jk}^v is a binary variable that indicates whether vehicle $v \in V$ travels from location $j \in S^+$ to location $k \in S^+$ or not.
- Z_{jk}^v is the cumulative demand rate of all remaining customers in a tour done by vehicle $v \in V$ when going from location $j \in S^+$ to location $k \in S^+$. It is zero if the vehicle does not go directly from j to k. This additional variable is needed to impose that tours start and end in the depot.

The first constraint, (2.12), imposes that a customer is visited by one and only one vehicle. Constraint (2.13) is the vehicle flow conservation constraint: the number of vehicles entering a location is equal to the number of vehicles leaving that location. Constraint (2.14) indicates that a vehicle is being used (and thus the fixed vehicle cost ψ has to be paid) as soon as it leaves the depot. The following two constraints impose the minimal cycle time, based on both the travelling, loading and unloading times (Constraint (2.15)) and on the customer imposed frequency constraints (Constraint (2.16)). Constraint (2.17) is the flow conservation constraint for the Z_{jk}^v variables: when visiting location k, the cumulative demand rate of all remaining customers in the tour is reduced by the demand rate of this location, d_k. Constraint (2.18) links the continuous Z_{jk}^v variables with the binary X_{jk}^v variables. The two final constraints determine

Table 2.3: Mixed integer model for cyclic inventory routing with multi-tours

$$\text{Min Z} = \sum_{v \in V} \left[\psi Y_v + \sum_{j \in S^+} \sum_{k \in S^+} X_{jk}^v \left(\frac{1}{T_v}(c_{jk} + \varphi_j) + T_v \frac{\eta_j d_j}{2} \right) \right]$$

subject to:

$$\sum_{v \in V} \sum_{j \in S^+} X_{jk}^v = 1 \qquad \forall k \in S \tag{2.12}$$

$$\sum_{j \in S^+} X_{jk}^v = \sum_{l \in S^+} X_{kl}^v \qquad \forall k \in S^+, v \in V \tag{2.13}$$

$$X_{\Delta j}^v \leq Y_v \qquad \forall j \in S, v \in V \tag{2.14}$$

$$\sum_{j \in S^+} \sum_{k \in S^+} X_{jk}^v (t_{jk} + t_j) \leq T_v \qquad \forall v \in V \tag{2.15}$$

$$\sum_{k \in S^+} X_{jk}^v \leq f_j T_v \qquad \forall j \in S, v \in V \tag{2.16}$$

$$\sum_{v \in V} \sum_{j \in S^+} Z_{jk}^v = d_k + \sum_{v \in V} \sum_{l \in S^+} Z_{kl}^v \qquad \forall k \in S \tag{2.17}$$

$$Z_{jk}^v \leq \left(\sum_{l \in S} d_l \right) X_{jk}^v \qquad \forall j, k \in S, v \in V \tag{2.18}$$

$$T_v Z_{\Delta j}^v \leq \kappa \qquad \forall j \in S, v \in V \tag{2.19}$$

$$T_v d_j \sum_{k \in S^+} X_{jk}^v \leq \kappa_j \qquad \forall j \in S, v \in V \tag{2.20}$$

$$T_v, Z_{jk}^v \geq 0, X_{jk}^v, Y_v \in \{0, 1\} \qquad \forall j, k \in S^+, v \in V$$

the maximal cycle time, based on both the vehicle capacity (Constraint (2.19)) and the customer storage capacities (Constraint (2.20)).

2.2.1 Multi-tours under driving time restrictions

When driving time restrictions are imposed, the minimal and maximal cycle time of a multi-tour have to be expressed as an integer number of days. For the maximal cycle time T_{MAX}, this is done by simply extending the formula for T_{max}.

$$T_{MAX} = \min\left(\min_{i=1..n}\left\lfloor\frac{\kappa}{\sum_{j\in S_i} d_j}\right\rfloor,\ \min_{i=1..n}\min_{j\in S_i}\left\lfloor\frac{\kappa_j}{d_j}\right\rfloor\right) \tag{2.21}$$

Determining the minimal cycle time T_{MIN} of a multi-tour under driving time restrictions is more complicated. The different tours that a vehicle makes have to be assigned to days in the cycle such that the driving time of the vehicle is less than 8 hours on any given day. The minimum number of days for which this can be done determines the minimal cycle time T_{MIN}. If all tours are between 4 and 8 hours long, no two tours can be made on the same day by the same vehicle without violating the 8-hour constraint and the number of days needed in the cycle is equal to the number of tours n. If there are also tours shorter than 4 hours, combining more than one tour on a single day may be feasible and the minimal cycle time T_{MIN} may reduce. As such, the number of tours n is an upper bound on the minimal cycle time.

As a result, the minimal cycle time T_{MIN} of a multi-tour under driving time restrictions is not given by a closed formula, but requires solving a mathematical programming problem. The model for this problem is given below. In this model, the parameter TD_i represents the time needed to finish tour i (i.e. $TD_i = t_\Delta + T_{TSP(S_i+\Delta)} + \sum_{j\in S_i} t_j$) and the binary variable X_i^t is used to indicate whether tour i is made on day t or not ($i \in 1..n$; $t \in 1..n$).

$$\text{Minimize } T_{MIN}$$

subject to:

$$\sum_{t=1}^{n} X_i^t = 1 \qquad i = 1..n \tag{2.22}$$

$$\sum_{i=1}^{n} TD_i X_i^t \leq 8 \qquad t = 1..n \tag{2.23}$$

$$\sum_{t=1}^{n} t \cdot X_i^t \leq T_{MIN} \qquad i = 1..n \tag{2.24}$$

$$X_i^t \in \{0,1\} \qquad i,t = 1..n$$

Constraint (2.22) imposes that all tours have to be made once during the cycle. Constraint (2.23) is the driving time restriction of 8 hours per day. Constraint (2.24), finally, defines the minimal cycle time, i.e. the minimal number of days needed to finish all tours. This problem is known in the literature as the Loading Problem [16]. Although its mathematical formulation is quite simple, this problem belongs to the class of NP-hard problems. Therefore, this problem (and its generalized version for distribution patterns) will be solved heuristically (see Section 3.6).

Illustrative example

When imposing the 8-hour per day driving constraint to our illustrative example, a feasible solution is only possible if customer 3 is replenished in a separate tour, because this separate tour takes exactly 8 hours. The following multi-tour solution with three tours is therefore suggested. The first tour visits customer 1 and takes 6 hours; the second tour visits customers 2 and 4 and takes 7 hours; the third tour goes to customer 3 and takes 8 hours. Because none of these tours can be combined in a single day, the minimal cycle time T_{MIN} is 3 days. The maximal cycle time of this multi-tour is $\min(120/(8 \cdot 4), 120/((8 \cdot 5), 120/(8 \cdot 1)) = 3$ days.

A cycle time of 3 days is therefore the only feasible cycle time for this multi-tour. The resulting cost rate is $C = 8\psi + \frac{50 \cdot 21}{T} + T \cdot (8 \cdot 0.15) \cdot (8 \cdot 5) = 160 + 350 + 144 = 654$ euro per day. This is more expensive than both multi-tour solutions presented above. Of course, the driving time restriction is the main reason for this, since it restricts the freedom in combining customers into efficient tours. This is also illustrated by the capacity utilization of only 67% (see Table 2.4), which is very poor because the vehicle is only filled with 24 units or 20% when making the tour to customer 3.

Table 2.4: Characteristics of the multi-tour solution under driving time restrictions

T_{MIN}	3 days
T_{MAX}	3 days
T_{EOQ}	5 days
Cycle time	3 days
q_1	96 units
q_2	72 units
q_3	24 units
q_4	48 units
Transport cost rate	350 €/day
Holding cost rate	144 €/day
Cost rate	654 €/day
Idle time	3/24 = 12.5%
Vehicle occupation	87.5%
Capacity utilization	66.7%

2.3 Modelling with more complex distribution patterns

By using the concept of a multi-tour, the same vehicle capacity can be used more than once to replenish a set of customers. This offers significantly more flexibility compared to single tours, where a vehicle capacity is used only once for as many customers as possible. However, more flexibility is still needed. In a multi-tour, the amount of time between two consecutive deliveries is the same for all customers that are being served by a single vehicle, regardless of the demand rate of the customer. This amount of time is the cycle time of the multi-tour. This means that a customer with a high demand rate is replenished just as often as a customer with a low demand rate. To find a better balance between distribution and holding costs, we feel that the customer with the high demand rate should be replenished more often than the customer with low demand rates. The extra flexibility that we need should allow a single vehicle to visit some customers more often than others, in order to obtain a better cost trade-off.

Suppose a set of customers is divided into two subsets, such that the cumulative demand rate of the customers in the first subset is about twice the cumulative demand rate of the customers in the second subset. Then the vehicle should make the tour to the first subset twice as often as the tour to the second subset. To obtain this, we introduce tour frequencies into the multi-tour concept. This generalized concept of a multi-frequency multi-tour, which we will call a 'distribution pattern', offers the flexibility that we need and thus the opportunity to obtain better cost trade-offs.

In a distribution pattern, a set of tours $T_1, \ldots, T_n$ is repeatedly made by the same vehicle, but these tours can be made a different number of times per cycle. Frequencies $k_1, \ldots, k_n$ are therefore introduced, meaning that tour T_i is made k_i times per cycle.

Lower bounds for the cycle time of the distribution pattern are given by (i) the total time needed to finish all tours the appropriate number of times, and (ii) the customer imposed delivery frequencies. Note that customer j in subset S_i is now replenished k_i times per cycle.

$$T_{min} = \max\left(\sum_{i=1}^{n} k_i \left(t_\Delta + T_{TSP(S_i+\Delta)} + \sum_{j\in S_i} t_j\right),\ \max_{i=1..n} \max_{j\in S_i} \frac{k_i}{f_j}\right) \tag{2.25}$$

During one cycle, tour i is made k_i times, so the vehicle can deliver at most k_i full truckloads κ to the customers in this tour. The cycle time of the distribution pattern can thus be at most $k_i\kappa/\sum_{j\in S_i} d_j$. Limited storage capacities of the visited customers also give an upper bound on the cycle time. The maximal cycle time T_{max} of the distribution pattern is then as follows.

$$T_{max} = \min\left(\min_{i=1..n} \frac{k_i\kappa}{\sum_{j\in S_i} d_j}, \min_{i=1..n}\min_{j\in S_i} \frac{k_i\kappa_j}{d_j}\right) \tag{2.26}$$

The extra flexibility offered by distribution patterns does not come without its price. Because the different tours the vehicle makes can have different frequencies, a schedule now needs to be constructed to put the tours in an acceptable sequence. Suppose e.g. a distribution pattern that has three tours with different frequencies: $k_1 = 4$, $k_2 = 2$ and $k_3 = 1$. Then it does not make sense to first make 4 times the first tour, then two times the second tour and then the last tour once (sequence $1-1-1-1-2-2-3$). Instead, the different iterations of the different tours will be interleaved to obtain a more favourable sequence, such as $1-2-1-3-1-2-1$.

The nature of this scheduling problem arising in designing distribution patterns depends on two problem characteristics. If it is obligatory that the time between consecutive deliveries is constant, the scheduling problem is completely different than if this is not the case. Viswanathan and Mathur (1997) [30] talk about a 'stationary' policy when deliveries are equidistant, but we do not adopt this term. The term 'stationary' is usually used to indicate the behaviour of a system parameter (e.g. stationary demand) and therefore we believe that using this term for solution concepts could confuse the reader. Instead, we will call distribution patterns with equidistant deliveries 'regular' and distribution patterns with non-equidistant deliveries 'irregular'. The second problem characteristic that results in a different scheduling problem, is the presence of driving time restrictions. The following paragraphs discuss the different scheduling problems arising in the different possible situations.

2.3.1 Regular distribution patterns

In a regular distribution pattern, the time between consecutive deliveries to a customer has to be constant. This means that the time between consecutive iterations of a tour also has to be constant. Consider e.g. a distribution pattern with two tours: the first tour, which takes 5 hours, has frequency 2, and the second tour, which takes 8 hours, has frequency 1. The minimal cycle time T_{min} of this distribution pattern is 18 hours, but it is impossible to make the first tour once every 9 hours, because every second time, the second tour has to be made in between. The minimal cycle time of 18 hours is thus infeasible. For feasibility, the minimal time between two iterations of the first tour has to be 13 hours ($= 5 + 8$), meaning that the actual lower bound on the cycle time is 26 hours.

This simple example shows that the restriction of equidistant deliveries introduces idle time into the routing scheme of a vehicle. Every now and then, the vehicle has to wait before it can start its next tour. To keep the interval of feasible cycle times as large as possible, a feasible routing scheme with a minimum of idle time has to be constructed. Thus, in the subproblem of scheduling the tours in a regular distribution pattern, our objective is to minimize the makespan of the schedule while maintaining constant times between consecutive iterations of the same tour. This minimal makespan is called the 'schedule time' and denoted T_{sched}. The interval of feasible cycle times is then $[\max(T_{min}, T_{sched}), T_{max}]$.

To mathematically model this scheduling problem, the following notation is used.

- $K = \sum_{i=1}^{n} k_i$: the total number of tours to be made in the schedule.
- $TD_i = t_\Delta + T_{TSP(S_i+\Delta)} + \sum_{j \in S_i} t_j$: duration of tour $i \in 1..n$.

In the schedule, K tours are made. The problem is to determine in which order the tours have to be made. The variables that are needed for the mathematical model are:

- X_{il}^k: binary variable indicating whether the l'th iteration of tour T_i is in k'th position in the sequence ($i \in 1..n$; $l \in 1..k_i$; $k \in 1..K$).
- t_{il}^k: starting time of the l'th iteration of tour T_i if it is in k'th position in the sequence, 0 otherwise ($i \in 1..n$; $l \in 1..k_i$; $k \in 1..K$).
- T_{sched}: makespan of the schedule.

The mathematical model for the tour scheduling problem in a regular distribution pattern is shown in Table 2.5.

The objective is to minimize the makespan of the schedule, while minimizing a number of constraints. Constraint (2.27) states that only one tour can be in k'th position of the sequence, while Constraint (2.28) states that the l'th iteration of tour i has to appear once in the sequence. This also means that the variable t_{il}^k is non-zero for only one value of k, which is imposed by Constraint (2.29). Constraints (2.30) fix a starting point in the cyclic schedule, without loss of generality. Constraint (2.31) indicates that a tour can only be started when the preceding tour is finished, and Constraint (2.32) indicates that the last tour in the sequence has to be finished within the schedule time T_{sched}. Constraint (2.33) ensures that deliveries are equidistant such that the distribution pattern is regular.

Solving the problem of scheduling a regular distribution pattern is computationally very complex. Since it contains a generalized assignment problem, it is in fact an NP-hard problem. Even for our small computational example below with only 4 tours with different frequencies, about half a minute is needed to solve the scheduling problem to optimality. We are well aware that the formulation of the problem given here can be significantly improved, and interesting valid inequalities can be added. However, this is beyond the scope of this

Table 2.5: Mathematical model for regular distribution pattern scheduling

$$\text{Minimize } T_{sched}$$

subject to:

$$\sum_{i=1}^{n}\sum_{l=1}^{k_i} X_{il}^{k} = 1 \qquad k = 1..K \tag{2.27}$$

$$\sum_{k=1}^{K} X_{il}^{k} = 1 \qquad \begin{cases} i = 1..n \\ l = 1..k_i \end{cases} \tag{2.28}$$

$$t_{il}^{k} \leq T_{max} X_{il}^{k} \qquad \begin{cases} i = 1..n \\ l = 1..k_i \\ k = 1..K \end{cases} \tag{2.29}$$

$$X_{11}^{1} = 1, \quad t_{11}^{1} = 0 \tag{2.30}$$

$$\sum_{i=1}^{n}\sum_{l=1}^{k_i} t_{il}^{k+1} \geq \sum_{i=1}^{n}\sum_{l=1}^{k_i} \left(t_{il}^{k} + TD_i \cdot X_{il}^{k}\right) \qquad k = 1..K-1 \tag{2.31}$$

$$T_{sched} \geq \sum_{i=1}^{n} \left(t_{ik_i}^{K} + TD_i \cdot X_{ik_i}^{K}\right) \tag{2.32}$$

$$\sum_{k=1}^{K} (t_{i(l+1)}^{k} - t_{il}^{k}) = \frac{T_{sched}}{k_i} \qquad \begin{cases} i = 1..n \\ l = 1..k_i - 1 \end{cases} \tag{2.33}$$

$$T_{sched}, t_{il}^{k} \geq 0, X_{il}^{k} \in \{0, 1\} \qquad \begin{cases} i = 1..n \\ l = 1..k_i \\ k = 1..K \end{cases} \tag{2.34}$$

work. Because of the computational complexity, a heuristic for this scheduling subproblem is developed in Section 3.6.

The formula for the cost rate of a regular distribution pattern is obtained from the formula for multi-tour cost rates by introducing the delivery frequencies k_i.

$$C = \psi + \frac{1}{T}\sum_{i=1}^{n} k_i \left(\varphi_\Delta + C_{TSP(S_i+\Delta)} + \sum_{j\in S_i} \varphi_j \right) + \frac{T}{2}\sum_{i=1}^{n}\sum_{j\in S_i} \frac{\eta_j d_j}{k_i} \tag{2.35}$$

A regular distribution pattern has a theoretical optimal cycle time, minimizing the cost rate. It is given by the following formula.

$$T_{eoq} = \left(\frac{\sum_{i=1}^{n} k_i \left(\varphi_\Delta + C_{TSP(S_i+\Delta)} + \sum_{j\in S} \varphi_j \right)}{\sum_{i=1}^{n}\sum_{j\in S_i} \frac{\eta_j d_j}{2k_i}} \right)^{1/2} \tag{2.36}$$

Illustrative example

For the illustrative 4-customer example, we consider the distribution pattern in which all customers are in a separate tour. If all these tours have frequency 1, the minimal cycle time is 22 hours, while the maximal cycle time is 30 hours. This solution is feasible, but it is not a good solution because the vehicle load is only 30 items (or 25% of the vehicle capacity) for the tour to customer 3.

To use vehicle capacity more efficiently and thus obtain a better solution, the frequencies of the tours are adapted. With frequencies proportional to the demand rates, the vehicle always leaves the depot with a full truckload. This means that the frequencies should be 4 for customer 1, 3 for customer 2, 1 for customer 3 and 2 for customer 4. This solution has a travel time of 52 hours ($= 4 \cdot 6 + 3 \cdot 4 + 8 + 2 \cdot 4$), and a maximal cycle time of 120 hours ($= \min(480/4, 360/3, 120/1, 240/2)$). The schedule for this regular distribution pattern with minimal makespan takes 120 hours (see Figure 2.3), which happens to be exactly the maximal cycle time. During these 120 hours, the vehicle is travelling only 52 hours. This distribution pattern is thus feasible if the cycle time is 120 hours. The cost rate is then $C = \psi + \frac{50\cdot 52}{T} + T \cdot 0.15 \cdot 2 = 20.00 + 21.67 + 36.00 = 77.67$ euro per hour.

When comparing this full-truckload distribution pattern as shown in Figure 2.3 to the multi-tour solution presented above (see Figure 2.2), it can be seen that the stock levels at the customers are much higher. Indeed, in the multi-tour solution, the best cycle time of 24 hours was below the EOQ cycle time, giving a cost balance in which distribution costs are higher than holding costs. In this full-truckload distribution pattern, it is the other way around. The best feasible cycle time of 120 hours is above the EOQ cycle time, such that the holding costs are now higher than the distribution costs. Below, other solutions are presented that give better cost trade-offs.

Within each cycle, the following happens. At time $t = 0$, the vehicle leaves to deliver a full truckload to customer 1. After 6 hours, the vehicle returns to the depot, is filled and leaves fully loaded for customer 2. Four hours later, the vehicle is in the depot again and a full truckload is dispatched for customer 3. When the vehicle returns from this tour, it has to wait for 12 hours, until time $t = 30$, when the tour to customer 1 is done for the second time. Six hours later, the vehicle returns and immediately makes the tour to customer 4. After this tour and 6 hours of waiting, the tour to the second customer is made for the second time. Then, the vehicle has to wait for 10 hours in the depot before making the tour to customer 1 for the third time. After another 20 hours of waiting, customer 2 is visited for the third time, immediately followed by the fourth visit to customer 1 and the second visit to customer 4. The cycle ends with another 20 hours of waiting time in the depot.

Table 2.6 gives an overview of the main characteristics of this full-truckload distribution pattern.

Table 2.6: Characteristics of the full-truckload distribution pattern

T_{min}	52h
T_{sched}	120h
T_{max}	120h
T_{eoq}	93.1h
Cycle time	120h
q_1	120 units
q_2	120 units
q_3	120 units
q_4	120 units
Transport cost rate	21.67 €/h
Holding cost rate	36.00 €/h
Cost rate	77.67 €/h
Idle time	68/120 = 56.7%
Vehicle occupation	43.3%
Capacity utilization	100%

Due to the incompatibility in the frequencies of customers 1 and 2, a lot of idle time is necessary to obtain a feasible schedule. As a consequence, the optimal cycle time of 93.1 hours is infeasible and this regular distribution pattern turns out to be even more expensive than the cheapest multi-tour solution.

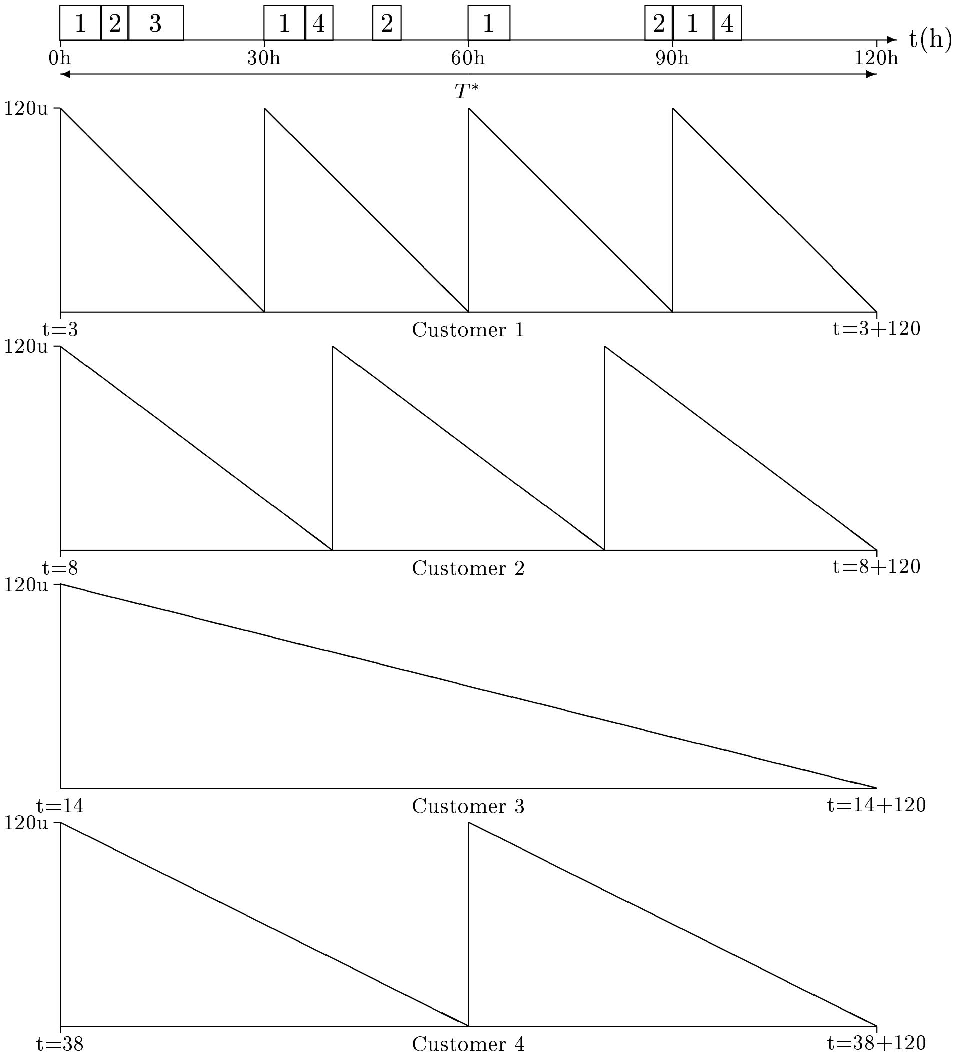


Figure 2.3: Schedule and stock levels for the full-truckload distribution pattern

Choosing the right tour frequencies

Every tour has its own optimal cycle time (see Section 2.1). In a distribution pattern, the tour frequencies should therefore be chosen such that the resulting cycle times for the tours are as close as possible to their respective optimal cycle times. If we denote the optimal tour cycle times as T_i^*, then we are looking for frequencies k_i and a cycle time T such that $T/k_i \approx T_i^*$.

Looking at the 4 tours in our illustrative example, the best feasible cycle times for the individual tours are respectively 30, 29.8, 73.0 and 36.5 hours. The following sets of values for the k_i's then seem appropriate: $(3,3,1,2)$ and $(2,2,1,2)$.

The cheapest solution is obtained when adopting the first of these possibilities: $k_1 = k_2 = 3, k_3 = 1$ and $k_4 = 2$. This gives a distribution pattern with a travel time of 46 hours, and a maximal cycle time of 90 hours. The schedule time is 66 hours (see Figure 2.4). The optimal cycle time of this distribution pattern is: $T_{eoq} = \sqrt{\frac{50 \cdot 46}{0.15 \cdot 13/6}} = 84.1$ hours. The cost rate obtained with a rounded cycle time of 84 hours is $C = \psi + \frac{50 \cdot 46}{T} + T \cdot 0.15 \cdot 13/6 = 20.00 + 27.38 + 27.30 = 74.68$ euro per hour.

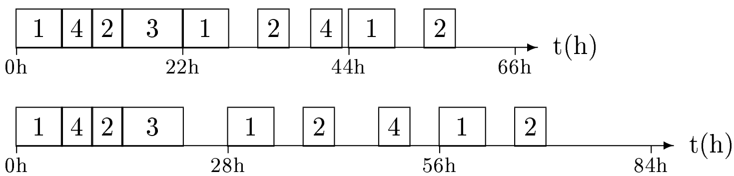


Figure 2.4: Schedule for the distribution pattern with adjusted frequencies

This is what happens during one cycle. The vehicle first delivers 112 units to customer 1, followed by a delivery of 84 units to customer 4, a delivery of 84 units to customer 2, and a delivery of 84 units to customer 3. Then there is 6 hours of idle time, after which the tours to customer 1 and customer 2 are repeated with 4 hours in between. After this, there is again 6 hours of idle time before the tour to customer 4 is repeated. Finally, the tours to customers 1 and 2 are made once more.

Table 2.7 gives an overview of the main characteristics of this solution.

In this solution, the differing tour frequencies still result in a large amount of idle time. However, the optimal cycle time is now feasible, such that the resulting cost rate is cheaper than that of the full-truckload distribution pattern and the multi-tour solutions presented above.

Powers-of-two

In the regular distribution patterns proposed above for the 4-customer example, a lot of idle time is unavoidable because the frequencies of customers 1, 2 and

Table 2.7: Characteristics of the distribution pattern with adjusted frequencies

T_{min}	46h
T_{sched}	66h
T_{max}	90h
T_{eoq}	84.1h
Cycle time	84h
q_1	112 units
q_2	84 units
q_3	84 units
q_4	84 units
Transport cost rate	27.38 €/h
Holding cost rate	27.30 €/h
Cost rate	74.68 €/h
Idle time	38/84 = 45.2%
Vehicle occupation	54.8%
Capacity utilization	77.8%

4 are 'incompatible'. To alleviate this, we could restrict the tour frequencies to a limited set of 'compatible' values. The most obvious approach is to restrict frequencies to powers-of-two, such that for every two possible frequencies, one is always an integer multiple of the other. Using this set of compatible frequencies will limit the need for idle time in the schedules.

Powers-of-two frequencies are often used in scheduling multiple products on a single machine (see e.g. Hahm and Yano, 1995 [21]). For these lot-sizing problems, it has been shown that powers-of-two frequency policies are within 6% of optimality.

When using powers-of-two frequencies, the regular scheduling problem can be reformulated. All frequencies k_i can now be written as 2^{m_i}, with $m_i \in \mathbb{N}^+$. The highest power-of-two, $\max_i(m_i)$ is denoted M. All tours have a frequency that is a divisor of the highest frequency 2^M. As a result, the schedule can be considered as a collection of 2^M 'buckets'. In each bucket, the tours that have the highest frequency are made together with some other tours with a lower frequency. The schedule is regular if tours are assigned to equidistant buckets. Consider e.g. a tour with frequency 4 when the highest frequency is 8. Then this tour has to reappear every second bucket to obtain a regular schedule, so the possible assignments are to buckets $1-3-5-7$ or $2-4-6-8$.

In Table 2.8, a mathematical model is given for assigning tours to buckets with the objective of obtaining a regular schedule with minimal makespan. In this model, the following variables are used:

- X_i^m: binary variable indicating whether the tour to S_i is included in the m'th bucket ($i \in 1..n$; $m \in 1..2^M$).
- L_m: length of the m'th bucket ($m \in 1..2^M$)

Table 2.8: Regular scheduling with powers-of-two

$$\text{Minimize } 2^M L_1$$

subject to:

$$\sum_{m=1}^{2^{M-m_i}} X_i^m = 1 \qquad i = 1..n \tag{2.37}$$

$$X_i^m = X_i^{m+k\cdot 2^{M-m_i}} \qquad \begin{cases} i = 1..n \\ m = 1..2^{M-m_i} \\ k = 1..2^{m_i} - 1 \end{cases} \tag{2.38}$$

$$L_m = \sum_{i=1}^{n} (TD_i \cdot X_i^m) \quad m = 1..2^M \tag{2.39}$$

$$L_1 \geq L_m \qquad m = 2..2^M \tag{2.40}$$

$$L_m \geq 0, X_i^m \in \{0,1\} \quad \begin{cases} i = 1..n \\ m = 1..2^M \end{cases}$$

Constraint (2.37) determines the first bucket to which a tour has to be assigned. Constraint (2.38) imposes that the assignment of tours to buckets is equidistant, making sure that the schedule is regular. Constraint (2.39) defines the length of the buckets and Constraint (2.40) states that the first bucket should be the longest of all. This can be done without loss of generality. Since the makespan of the schedule has to be 2^M times the length of the longest bucket, the objective is to minimize the length of the first, longest bucket.

This reformulation of the regular scheduling problem for the case of powers-of-two frequencies is still computationally very complex. In fact, it is still NP-complete. Since the computational burden is still too high, the same heuristic from Section 3.6 will be used regardless of whether the frequencies are powers-of-two or not.

If only powers-of-two are allowed for the tour frequencies in our illustrative example, shown again in Figure 2.5, the appropriate frequencies are 2, 2, 1 and 2 respectively. This solution has a travel time of 36 hours, and a maximal cycle time of 60 hours ($= \min(240/4, 240/3, 120/1, 240/2)$). The regular schedule with minimal makespan takes 44 hours (see Figure 2.6), so there is indeed much less idle time compared to the distribution patterns presented above.

The optimal cycle time of this distribution pattern with power-of-two frequencies is: $T_{eoq} = \sqrt{\frac{50\cdot 36}{0.15\cdot 11/4}} = 66.1$ hours. This is not a feasible cycle time. The resulting cost rate at the best feasible cycle time of 60 hours is: $C = \psi + \frac{50\cdot 36}{T} + T \cdot 0.15 \cdot 11/4 = 20.00 + 30.00 + 24.75 = 74.75$ euro per hour. This powers-of-two solution is cheaper than the full-truckload distribution pat-

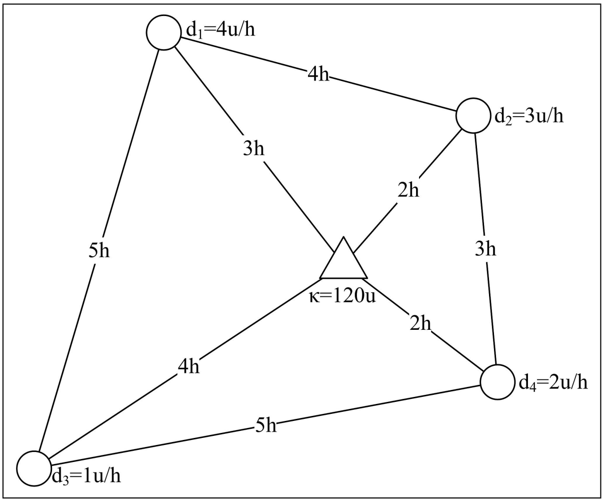


Figure 2.5: Illustrative 4-customer example

tern because we can get closer to the optimal cycle time, and thus a more favourable cost trade-off can be obtained. However, it is still more expensive than the distribution pattern with frequencies $(3, 3, 1, 2)$.

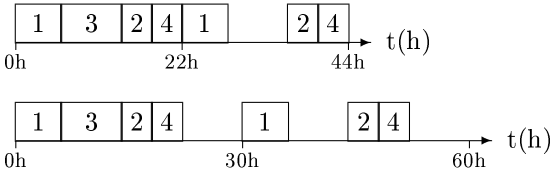


Figure 2.6: Minimal-makespan and actual schedule for the powers-of-two distribution pattern

In one iteration of the cycle, the following happens. The vehicle first makes the tour to customer 1 with a load of 120 units, immediately followed by the tour to customer 3 with a load of 60 units, the tour to customer 2 with a load of 90 units and the tour to customer 4 with a load of 60 units. Then, the vehicle has 8 hours of idle time, before it repeats the tours to customer 1, and the tours to customers 2 and 4.

Table 2.9 gives an overview of the main characteristics of this powers-of-two distribution pattern.

Optimal solution for the illustrative example

The optimal solution for our example has not been encountered so far. It is a distribution pattern consisting of three tours. Customer 1 is replenished

Table 2.9: Characteristics of the powers-of-two distribution pattern

T_{min}	36h
T_{sched}	44h
T_{max}	60h
T_{eoq}	66.1h
Cycle time	60h
q_1	120 units
q_2	90 units
q_3	60 units
q_4	60 units
Transport cost rate	30.00 €/h
Holding cost rate	24.75 €/h
Cost rate	74.75 €/h
Idle time	$24/60 = 40\%$
Vehicle occupation	60%
Capacity utilization	71.43%

by the first tour, which has frequency 3. Customers 2 and 4 are replenished together in a second tour, that also has frequency 3. Customer 3, finally, is replenished separately in a tour with frequency 1. This solution has a travel time of $3 \cdot 6 + 3 \cdot 7 + 8 = 47$ hours, and a maximal cycle time of 72 hours $(= \min(360/4, 360/5, 120/1))$. The minimal makespan for a regular schedule is 63 hours. This schedule is shown in Figure 2.7. In Section 3.6, we will show how this and all other schedules are constructed using the heuristic that is developed there.

The optimal cycle time of this distribution pattern is: $T_{eoq} = \sqrt{\frac{50 \cdot 47}{0.15 \cdot 2}} = 88.5$ hours. This is infeasible, so the best feasible cycle time is 72 hours, resulting in the following cost rate: $C = \psi + \frac{50 \cdot 47}{T} + T \cdot 0.15 \cdot 2 = 20.00 + 32.64 + 21.60 = 74.24$ euro per hour.

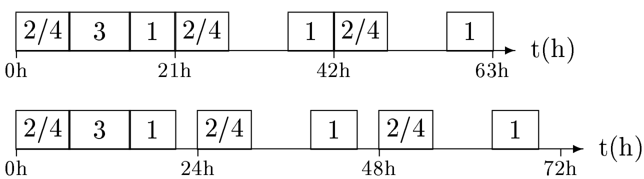


Figure 2.7: Minimal-makespan and actual schedule for the optimal distribution pattern

During one iteration of this distribution pattern, the following events take place. At time $t = 0$, the vehicle leaves the depot fully loaded to make the tour to customers 2 and 4. After 2 hours, the vehicle arrives at customer 2 and delivers 72 units, and after 5 hours, the vehicle delivers the remaining 48 units to customer 4. Two hours later, the vehicle is back in the depot. It is then reloaded with 72 units that are delivered to customer 3 after 11 hours. Four hours later, the vehicle is back in the depot, where it is loaded with 96 units destined for customer 1. This delivery occurs at time $t = 18$. The vehicle then returns to the depot and waits for 3 hours before making the first and third tour again, without repeating the second tour in between. Then, the first and third tour are repeated once more.

Table 2.10 gives an overview of the main characteristics of the optimal distribution pattern.

Table 2.10: Characteristics of the optimal distribution pattern

T_{min}	47h
T_{sched}	63h
T_{max}	72h
T_{eoq}	88.5h
Cycle time	72h
q_1	96 units
q_2	72 units
q_3	72 units
q_4	48 units
Transport cost rate	32.64 €/h
Holding cost rate	21.60 €/h
Cost rate	74.24 €/h
Idle time	$25/72 = 34.7\%$
Vehicle occupation	65.3%
Capacity utilization	85.7%

2.3.2 Irregular distribution patterns

In a regular distribution pattern, a customer always receives the same quantities with a constant time between the deliveries. As a result, the stock level displays a perfectly regular jigsaw pattern. The formulas for the maximal cycle time (2.26), cost rate (2.35) and EOQ cycle time (2.36) are based on this property.

In an irregular distribution pattern, the restriction of equidistant deliveries is dropped. Customers can now receive different quantities with differing inter-delivery times, so the stock levels can show irregular jigsaw patterns. Therefore, the formulas for the maximal cycle time, cost rate and EOQ cycle time are no longer valid. Figure 2.8 shows an example of an irregular stock level pattern. A customer consuming 3 units per hour receives three deliveries in a 20-hour cycle. The delivery quantities q, q', q'' are the same, but the time between deliveries varies. As a result, stock builds up and two out of three times, the customer is replenished when there is still stock left ($s', s'' > 0$).

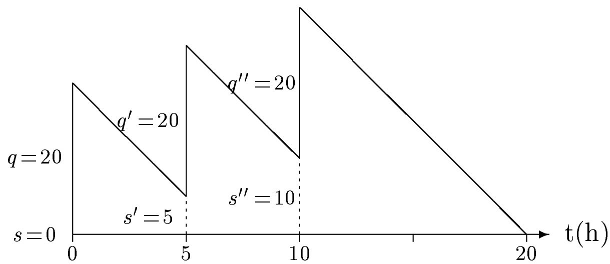


Figure 2.8: Irregular stock level pattern

In the cost rate of an irregular distribution pattern, the first four components are independent of the schedule, but the fifth component, the holding cost rate, does depend on the actual schedule and delivery quantities. For the fifth component, a closed expression can no longer be found. The holding cost component of Formula (2.35) gives a lower bound for the actual holding cost rate, and thus Formula (2.36) gives an upper bound for the optimal cycle time. To obtain the actual optimal cycle time and cost rate, an iterative procedure would be needed. Starting with a cycle time given by Formula (2.36), a schedule is constructed. From this schedule, the actual holding cost rate is determined. Adjusting the formula for the optimal cycle time with this actual holding cost can give a new cycle time. For this adjusted cycle time, a new schedule is constructed, and so on, until the cycle time converges.

For the problem of scheduling the tours in irregular distribution pattern, the mathematical model of Table 2.5 can be recycled. However, in the irregular case the makespan T_{sched} of the schedule is no longer a variable but a given parameter. The objective is now to minimize holding costs at the customers without violating any of the capacity restrictions. Therefore, the following notations are introduced.

- $\eta_i = \frac{\sum_{j \in S_i} \eta_j d_j}{\sum_{j \in S_i} d_j}$: the weighted average holding cost rate for the customers in S_i.
- q_{il}: variable that gives the vehicle load to be distributed over the customers in the l'th iteration of tour i ($i \in 1..n$; $l \in 1..k_i$).
- s_{il}: cumulative stock level of the customers in S_i at the time of the l'th delivery ($i \in 1..n$; $k \in 1..K$).

The mathematical model for the tour scheduling problem in an irregular distribution pattern is shown in Table 2.11.

Constraints (2.41) and (2.42) represent the flow of goods. Constraint (2.43) is the vehicle capacity constraint, and Constraint (2.44) the customer storage capacity constraint. Constraint (2.45), finally, imposes that the total delivered quantity covers the demand during one cycle.

Table 2.11: Mathematical model for irregular distribution pattern scheduling

$$\text{Minimize} \quad \frac{1}{T_{sched}} \sum_{i=1}^{n} \eta_i \left[\sum_{l=1}^{k_i-1} \sum_{k=1}^{K} (t^k_{i(l+1)} - t^k_{il}) \left(\frac{s_{il} + q_{il} + s_{i(l+1)}}{2} \right) + \sum_{k=1}^{K} (t^k_{i1} + T_{sched} - t^k_{ik_i}) \left(\frac{s_{ik_i} + q_{ik_i} + s_{i1}}{2} \right) \right]$$

subject to:

$$(2.27) - (2.32), (2.34)$$

$$s_{i(l+1)} = s_{il} + q_{il} - \sum_{k=1}^{K} (t^k_{i(l+1)} - t^k_{il}) \sum_{j \in S_i} d_j \qquad \begin{cases} i = 1..n \\ l = 1..k_i - 1 \end{cases} \tag{2.41}$$

$$s_{i1} = s_{ik_i} + q_{ik_i} - \sum_{k=1}^{K} (t^k_{i1} + T_{sched} - t^k_{ik_i}) \sum_{j \in S_i} d_j \qquad i = 1..n \tag{2.42}$$

$$q_{il} \leq \kappa \qquad \begin{cases} i = 1..n \\ l = 1..k_i \end{cases} \tag{2.43}$$

$$\frac{d_j}{\sum_{j' \in S_i} d_{j'}} (s_{il} + q_{il}) \leq \kappa_j \qquad \begin{cases} i = 1..n \\ l = 1..k_i \\ j \in S_i \end{cases} \tag{2.44}$$

$$\sum_{l=1}^{k_i} q_{il} = T \sum_{j \in S_i} d_j \qquad i = 1..n \tag{2.45}$$

$$q_{il}, s_{il} \geq 0 \qquad \begin{cases} i = 1..n \\ l = 1..k_i \end{cases}$$

The objective of the problem is to choose the delivery timing (t_{il}^k variables) and quantities (q_{il} variables) such that the total holding cost rate is minimized. The holding cost corresponds to the cumulative surface under the stock level curves (see e.g. Figure 2.8). The objective function is thus quadratic and the computational complexity of this scheduling problem extremely high. Therefore, as for scheduling regular distribution patterns, a heuristic is adopted for solving it (see Section 3.6).

Illustrative example

For our illustrative example, the regular full-truckload distribution pattern has a schedule time that is equal to the maximal cycle time of 120 hours, while the EOQ cycle time is 93.1 hours. If we now allow irregular solutions, this cycle time of 93.1 hours becomes feasible. However, to avoid working with fractional values, we round this cycle time to 96 hours. For the tours to customers 1, 3 and 4, it is possible to keep the deliveries equidistant. For the tour to customer 2, with the incompatible frequency, this is not the case. The second and third delivery have to be started a bit earlier, such that the time between the third iteration in a cycle and the first iteration in the next cycle is slightly larger than the time between other iterations. Figure 2.9 shows the schedule and the stock level of customer 2 with the cycle time rounded to 96 hours.

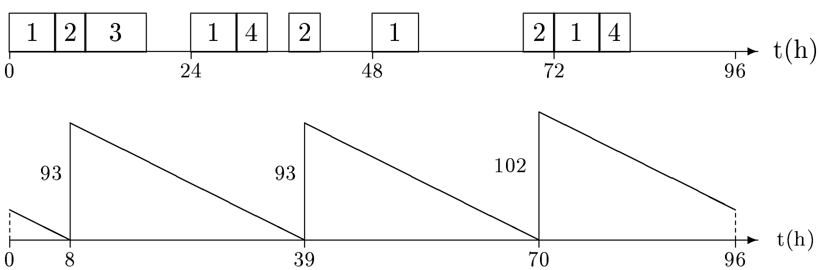


Figure 2.9: Schedule and stock level for the irregular distribution pattern

To have a regular distribution pattern, the vehicle should bring three times 96 units to customer 2, with 32 hours in between. This is infeasible, so the following situation, which is very close to being regular, is obtained. In the first two visits in a cycle, deliveries of only 93 units are made, so that there is 31 hours until the next delivery. The third and last delivery then has to cover 34 hours and therefore brings 102 units.

Table 2.12 gives an overview of the main characteristics of the irregular distribution pattern.

This solution is cheaper than the full-truckload and the powers-of-two distribution pattern, but it is still more expensive than the optimal distribution pattern, which is regular.

Table 2.12: Characteristics of the irregular distribution pattern

T_{min}	52h
T_{max}	120h
T_{eoq}	93.1h
Cycle time	96h
q_1	4 times 96 units
q_2	93, 93 and 102 units
q_3	96 units
q_4	2 times 96 units
Transport cost rate	27.08 €/h
Holding cost rate	28.81 €/h
Cost rate	75.89 €/h
Idle time	44/96 = 45.8%
Vehicle occupation	54.2%
Capacity utilization	80%

2.3.3 Distribution patterns under driving time restrictions

When driving time restrictions are imposed, the minimal and maximal cycle time of a distribution pattern have to be expressed as an integer number of days. For the maximal cycle time T_{MAX}, this is done by simply extending the formula for T_{max}.

$$T_{MAX} = \min\left(\min_{i=1..n}\left\lfloor\frac{k_i\kappa}{\sum_{j\in S_i} d_j}\right\rfloor, \min_{i=1..n}\min_{j\in S_i}\left\lfloor\frac{k_i\kappa_j}{d_j}\right\rfloor\right) \tag{2.46}$$

As for multi-tours, the minimal cycle time T_{MIN} is no longer given by a closed expression, but requires solving a mathematical programming problem. The different iterations of the different tours that a vehicle makes have to be assigned to days in the cycle such that the driving time of the vehicle is less than 8 hours on any given day. The minimum number of days for which this can be done determines the minimal cycle time T_{MIN}. If all tours are between 4 and 8 hours long, no two tours can be made on the same day by the same vehicle without violating the 8-hour constraint and the number of days needed in the cycle is equal to the total number of tours $K = \sum_i^n k_i$. If tours shorter than 4 hours exist, making more than one tour on a single day may be feasible and the minimal cycle time T_{MIN} may reduce. As such, K is an upper bound on the minimal cycle time.

The mathematical model for determining T_{MIN} is given below. In this model, the parameter TD_i again represents the time needed to finish tour i (i.e. $TD_i = t_\Delta + T_{TSP(S_i+\Delta)} + \sum_{j\in S_i} t_j$) and the binary variable X_{il}^t is used to indicate whether the l'th iteration of tour i is made on day t or not ($i \in 1..n$; $l \in 1..k_i$; $t \in 1..n$).

$$\text{Minimize } T_{MIN}$$

subject to:

$$\sum_{t=1}^{K} X_{il}^{t} = 1 \qquad i = 1..n, l = 1..k_i \tag{2.47}$$

$$\sum_{l=1}^{k_i} X_{il}^{t} \leq 1 \qquad i = 1..n, t = 1..K \tag{2.48}$$

$$\sum_{i=1}^{n} \sum_{l=1}^{k_i} TD_i X_{il}^{t} \leq 8 \quad t = 1..K \tag{2.49}$$

$$\sum_{t=1}^{n} t \cdot X_{il}^{t} \leq T_{MIN} \quad i = 1..n \tag{2.50}$$

$$X_{il}^{t} \in \{0, 1\} \qquad i, t = 1..n$$

Constraint (2.47) ensures that all iterations of each of the tours are made once during the cycle, while Constraint (2.47) imposes that not two iterations of the same tour are on the same day. Constraint (2.49) is the driving time restriction of 8 hours per day. Constraint (2.50), finally, defines the minimal cycle time, i.e. the minimal number of days needed to finish all iterations of all tours.

Determining the optimal cycle time and cost rate and developing the actual schedule for a distribution pattern under driving time restrictions again requires a different approach based on whether the customers impose equidistant deliveries or not.

Regular distribution patterns

When driving time restrictions apply, we assume that a tour cannot be made more than once per day. For regular distribution patterns, this means that the time between two iterations of a tour, T/k_i has to be an integer number of days. As a result, the cycle time T of the distribution pattern, expressed in number of days, has to be an integer multiple of all frequencies k_i. Thus, the cycle time is always an integer multiple of the least common multiple of all frequencies k_i. This least common multiple defines what we call the base cycle time B. E.g. if a distribution pattern makes two tours, one with frequency 2 and another with frequency 3, then the cycle time of the distribution pattern has to be a multiple of $B = 6$ days if equidistant deliveries are required.

If the vehicle can only drive on weekdays and not during the weekend, the cycle time of the distribution pattern also has to be an integer multiple of 5. Then, the time between consecutive deliveries to any customer is an integer number of weeks, which makes sure that no delivery ever occurs during weekends. In this case, days 6 and 7 in a schedule are not Saturday and Sunday, but Monday and Tuesday of the next week.

Based on this base cycle time B, the minimal and maximal cycle time can also be adjusted. In the mathematical model for determining the minimal cycle time T_{MIN}, the following constraint can be added.

$$T_{MIN} \bmod B = 0 \tag{2.51}$$

The formula for the maximal cycle time T_{MAX} changes as follows.

$$T_{MAX} = B \left\lfloor \frac{1}{B} \min \left(\min_{i=1..n} \frac{k_i \kappa}{\sum_{j \in S_i} d_j}, \min_{i=1..n} \min_{j \in S_i} \frac{k_i \kappa_j}{d_j} \right) \right\rfloor \tag{2.52}$$

Once a cycle time of a certain number of days is given, the tours in a distribution pattern have to be assigned to days such that (i) the vehicle drives less than a pre-specified number of hours (usually 8) per day, and (ii) the number of days between consecutive iterations of the same tour is constant. The scheduling problem arising here is thus an extended version of the scheduling problem for multi-tours. Table 2.13 gives a mathematical model for this problem.

In the model, the binary variable X_{il}^t indicates whether the l'th iteration of tour i is done on day t or not ($i \in 1..n$; $l \in 1..k_i$; $t \in 1..T_{MAX}$, while the integer variable T_{il} represents the day on which the l'th iteration of tour i is made ($i \in 1..n$; $l \in 1..k_i$). T_S is the schedule time expressed as a number of days. The objective is again to construct a schedule with a minimum idle time and thus with a minimal makespan.

Constraint (2.53) fixes a starting point in the cycle by stating that the first iteration of the first tour has to be made on the first day. Tour i has to be made k_i times during a cycle, and T_{MAX} is the upper bound on the cycle time. Thus, the first iteration of tour i should be on day $\frac{T_{MAX}}{k_i}$ at the latest, the second iteration on day $2\frac{T_{MAX}}{k_i}$ at the latest, and so on. Similarly, because of the lower bound T_{MIN} on the cycle time, the second iteration of tour i can be made on day $\frac{T_{MIN}}{k_i}+1$ at the earliest, the third iteration can be on day $2\frac{T_{MIN}}{k_i}+1$ at the earliest, and so on. This is imposed by Constraints (2.54), (2.55) and (2.56). Constraint (2.57) derives the values of the dummy T_{il} variables from the X_{il}^t variables. Constraint (2.58) imposes that deliveries are equidistant, such that the number of days between two iterations of the same tour is always the same, and Constraint (2.59) ensures that the last iteration of tour i is done within the time horizon T_S. Note that if T_S is smaller than T_{MAX}, all X_{il}^t-variables become zero for $t > T_{SCHED}$. The actual driving time restriction of 8 hours per day is given in Constraint (2.60). Constraint (2.61), finally, states that the cycle time has to be an integer multiple of the base cycle time B.

In this model, it is assumed that the minimal cycle time is known. However, this requires solving another, related model. To avoid this, the following lower bound on the cycle time T_M can be used instead of T_{MIN}.

Table 2.13: Regular scheduling with driving time restrictions

$$\text{Minimize } T_S$$

subject to:

$$X_{11}^1 = 1, \quad T_{11} = 1 \tag{2.53}$$

$$\sum_{t=1}^{(l-1)\frac{T_{MIN}}{k_i}} X_{il}^t = 0 \qquad \left\{ \begin{array}{l} i = 1..n \\ l = 2..k_i \end{array} \right. \tag{2.54}$$

$$\sum_{t=(l-1)\frac{T_{MIN}}{k_i}+1}^{l\frac{T_{MAX}}{k_i}} X_{il}^t = 1 \qquad \left\{ \begin{array}{l} i = 1..n \\ l = 1..k_i \end{array} \right. \tag{2.55}$$

$$\sum_{t=l\frac{T_{MAX}}{k_i}+1}^{T_{MAX}} X_{il}^t = 0 \qquad \left\{ \begin{array}{l} i = 1..n \\ l = 1..k_i \end{array} \right. \tag{2.56}$$

$$T_{il} = \sum_{t=1}^{T_{MAX}} t \cdot X_{il}^t \qquad \left\{ \begin{array}{l} i = 1..n \\ l = 1..k_i \end{array} \right. \tag{2.57}$$

$$T_{i,l+1} = T_{il} + T_S/k_i \qquad \left\{ \begin{array}{l} i = 1..n \\ l = 1..k_i - 1 \end{array} \right. \tag{2.58}$$

$$T_{ik_i} \leq T_S \qquad i = 1..n \tag{2.59}$$

$$\sum_{i=1}^{n} \sum_{l=1}^{k_i} \left(TD_i \cdot X_{il}^t \right) \leq 8 \qquad t = 1..T_{MAX} \tag{2.60}$$

$$T_S \bmod B = 0 \tag{2.61}$$

$$X_{il}^t \in \{0,1\}, T_{il} \in 1..T_{MAX} \quad \left\{ \begin{array}{l} i = 1..n \\ l = 1..k_i \\ t = 1..T_{MAX} \end{array} \right.$$

$$T_M = B \left\lceil \frac{1}{B} \max \left(\sum_{i=1}^{n} k_i \left(t_\Delta + T_{TSP(S_i+\Delta)} + \sum_{j \in S_i} t_j \right), \max_{i=1..n} \max_{j \in S_i} \frac{k_i}{f_j} \right) \right\rceil \tag{2.62}$$

Constraints (2.54), (2.55), (2.56) and (2.60) constitute a generalized assignment problem, which is known to be NP-hard [10]. This scheduling problem is thus also NP-hard. Therefore, a heuristic solution approach is developed in Section 3.6.

Illustrative example

If we assume that demand only occurs during the 8 working hours of a day, then from all proposed solutions for our illustrative example only two solutions remain valid when the driving time restriction is imposed.

Because of the frequencies of 4, 3, 1 and 2, the cycle time of the full-truckload distribution pattern has to be an integer multiple of 12 days or 96 hours. The only possible cycle time is 120 hours, which is not a multiple of 96 hours, so the full-truckload distribution pattern becomes infeasible.

In the 4-tour distribution pattern with adjusted frequencies, the least common multiples of the frequencies is 6, so the cycle time has to be a multiple of 48 hours. However, there is no multiple of 48 hours between 84 and 90 hours, which is the range of possible cycle times for this distribution pattern, so it becomes infeasible as well.

Next is the powers-of-two distribution pattern. The least common multiple of its frequencies is 2, so the cycle time has to be a multiple of 16 hours. The only feasible cycle time in the possible interval between 44 and 60 hours is at 48 hours, or 6 days. The resulting schedule is shown in Figure 2.10. The tours to customers 2 and 4 have to be made on the same day, which is perfectly possible because they take only 4 hours each. Because the cycle time changes, the cost rate also increases to 77.30 euro per hour, or 618.40 euro per day.

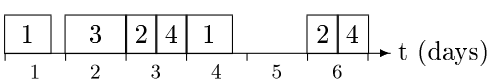


Figure 2.10: Schedule for the powers-of-two distribution pattern under driving time restrictions

The optimal solution is the other solution that is still feasible when the driving time restriction is imposed. The cycle time is 72 hours, which is exactly 9 days. This is an integer multiple of the least common multiple of all frequencies, which is 3. A schedule is shown in Figure 2.11. The tour to customer 1 is done on days 1, 4 and 7, the tour to customers 2 and 4 is done on days 2, 5 and 8 and the tour to customer 3 is done on day 3. The cost rate of this solution is still the same, 74.24 euro per hour, or 593.9 euro per day. The optimal solution thus remains optimal under the driving time restriction.

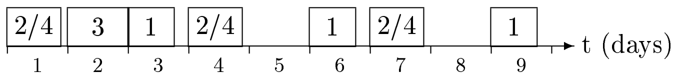


Figure 2.11: Schedule for the optimal distribution pattern under driving time restrictions

Irregular distribution patterns

For irregular distribution patterns under driving time restrictions, the cycle time does not have to be an integer multiple of the tour frequencies k_i, but merely an integer number of days. In the resulting scheduling problem, all iterations of each of the tours have to be assigned to a day in the cycle with the objective of minimizing the total holding cost at the customers. A mathematical model for this scheduling problem is obtained by discretizing the model of Table 2.11. However, the model is not presented here. As all other versions of the scheduling problem, this is an NP-hard problem and the heuristic procedure presented in Section 2.5 is used for solving it.

The full-truckload distribution pattern of our illustrative example has an optimal cycle time of 93.1 hours, which is close to 96 hours or 12 days. A possible schedule is shown in Figure 2.12. In this solution, the deliveries to customers 1, 3 and 4 are equidistant. The resulting stock levels for customer 2, who does not receive perfectly equidistant deliveries, are also shown in Figure 2.12. The cost rate of this full-truckload distribution pattern is 607.7 euro per day. It is thus cheaper than the regular full-truckload distribution pattern without driving time restrictions, which costs 77.67 euro per hour, or 621.4 euro per day, but still more expensive than the optimal, regular distribution pattern shown above.

Another regular distribution pattern solution that becomes infeasible under driving time constraints is the 4-tour distribution pattern with adjusted frequencies. When deliveries no longer have to be equidistant, it still gives a feasible irregular distribution pattern under driving time constraints. A possible schedule with a cycle time of 11 days is shown in Table 2.13. In this schedule, customers 3 and 4 still have equidistant deliveries, but customers 1 and 2 no longer have a perfect jigsaw stock level curve. Due to their increased holding cost rates, the cost rate of this distribution pattern increases from 597.4 to 608.6 euro per day.

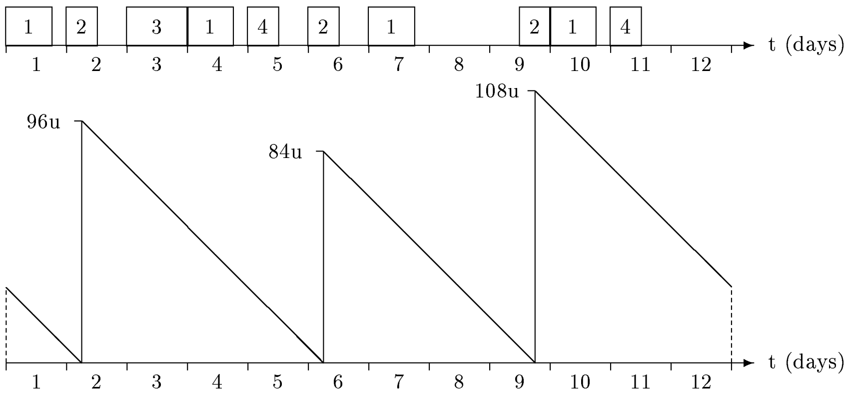


Figure 2.12: Irregular schedule for the full-truckload distribution pattern

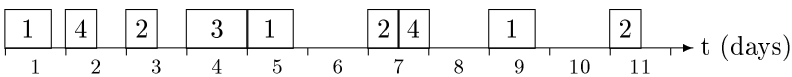


Figure 2.13: Schedule for another irregular distribution pattern

2.4 Conclusion

When using the concept of tours, it is assumed that a separate vehicle is available for each tour. In other words, the assignment of tours to vehicles is not considered. In long-term routing problems, the vehicle fleet becomes variable and a fixed cost per vehicle has to be included. As a result, the assignment of tours to vehicles can no longer be neglected. The concept of tours then becomes inadequate. In this chapter, we introduce a generalized routing concept, to be used in long-term routing problems where a cyclic solution approach is used. This generalized concept is the 'distribution pattern', a cyclic routing scheme for a single vehicle consisting of multiple tours that can be repeated with different frequencies.

In the concept of distribution patterns, many real-life characteristics and restrictions can be considered. A maximal cycle time is imposed resulting from the vehicle capacity and the limited customer storage capacities. A minimal cycle time is imposed, taking into account driving, loading and unloading times, but also customer visiting frequency restrictions.

Another important feature is the scheduling problem that arises in designing distribution patterns. The nature of this scheduling problem depends on the presence of two other restrictions. The first extra restriction that can be imposed is that deliveries have to be equidistant, resulting in so-called 'regular' distribution patterns. The second restriction is the driving time restriction, imposing that a vehicle cannot drive more than a certain number of hours (usually 8) per day, resulting in a discrete version of the scheduling problem. Although solution approaches for the different situations are presented in the next chapter, we will make the realistic assumptions in our computational experiments that deliveries have to be equidistant and driving times are restricted to 8 hours per day.

In distribution patterns, vehicle capacity is reused for multiple customer subsets, thus reducing the required vehicle fleet size. Furthermore, allowing different tours to be made by the same vehicle, possibly with different frequencies, gives the opportunity to obtain cycle times closer to the individual optimal tour cycle times, resulting in a better balance between distribution and holding costs. Distribution patterns are therefore the appropriate concept for the long-term, cyclic inventory routing problem in which a three-way cost trade-off has to be found between (i) fixed vehicle fleet costs, (ii) distribution costs, and (iii) stock holding costs.

Chapter 3

Solution approach

The problem of developing an infinite-horizon routing plan for the cyclic inventory routing problem with constant demand rates is highly complex. It can be considered as a combination of inter-related and nested subproblems. A possible top-down decomposition of the overall problem into subproblems is as follows.

1. Customer are assigned to vehicles, or, the set of customers is partitioned over a (variable) number of vehicles.
2. In a distribution pattern, a vehicle can make different tours. Therefore, the subset of customers assigned to a vehicle is further partitioned into sub-subsets. The customers in a sub-subset will then be replenished together in a separate tour.
3. For every customer sub-subset, a tour is constructed. This tour is the shortest depot-to-depot tour that visits all customers in the sub-subset. (This is the well-known Travelling Salesman Problem.)
4. The different tours that a vehicle makes can have different frequencies. Therefore, for each vehicle (i.e. for each distribution pattern), appropriate tour frequencies are determined.
5. For each distribution pattern, a delivery schedule is constructed to check feasibility, find the best feasible cycle time and determine the cost rate.

In this chapter, different solution approaches are presented that try to find good trade-offs between the different cost components, thus obtaining global cost minimizing solutions.

First, two constructive heuristics are proposed: an insertion heuristic in Section 3.1 and a savings heuristic in Section 3.2. Next, in Section 3.3, an improvement heuristic is suggested for the solutions generated by either of the constructive heuristics.

The proposed heuristics deal with the first two of the subproblems listed above, i.e. the assignment of customers to vehicles and tours. How the remaining subproblems are tackled within these heuristics, is explained in the subsequent sections of this chapter. Section 3.4 discusses the construction of tours, Section 3.5 deals with determining tour frequencies in a distribution pattern, and Section 3.6 finally, focuses on the problem of constructing delivery schedules for the vehicles.

For the long-term problem that we are studying, computation times are not restrictive. Since the proposed construction and improvement heuristics are relatively fast, they are adjusted to be iteratively used within metaheuristic frameworks: first in a multi-start procedure (Section 3.7), and next in a column generation procedure (Section 3.8).

3.1 An insertion heuristic

An elementary way of constructing a solution is to start with an empty solution and then insert customers into the solution one by one. Such a solution approach is called an 'insertion heuristic' [29]. In an insertion heuristic, the quality of the solution is highly dependent on the order in which customers are inserted into the solution. The idea that we adopt here, is to first insert customers that are expected to generate high costs and then fill up the solution with the less cost-critical customers. To do this, customers are sorted according to a certain priority. The priority that we propose for a customer is the cost rate of the distribution pattern that covers only this customer.

The proposed insertion heuristic then has the following steps.

1. Generate the 'basic' distribution patterns, each serving a single customer and use their cost rates for the customer priorities λ_j. Sort the customers according to this priority.

2. Initialize the solution with a single basic distribution pattern that covers the customer with the highest priority.

3. Insert the customer with the next highest priority λ_j into the solution. Consider the following alternatives: insert the customer into each of the existing distribution patterns or generate a new basic distribution pattern visiting only this customer. When inserting a customer into an existing distribution pattern, the following possibilities are available: the customer can be inserted into one of the tours of the distribution pattern, or a separate tour to this customer can be added to the distribution pattern. The cheapest of all alternatives is kept.

4. Repeat the previous step until all customers have been inserted into the solution.

When inserting a customer into an existing distribution pattern, the frequencies of the tours in this distribution pattern are being reconsidered, and new delivery schedules have to be constructed. The discussion of how these subproblems are being solved is presented later in this chapter, in Sections 3.5 and 3.6.

Illustrative example

To illustrate the insertion heuristic, it is applied to the same 4-customer example that we have used in the previous chapter.

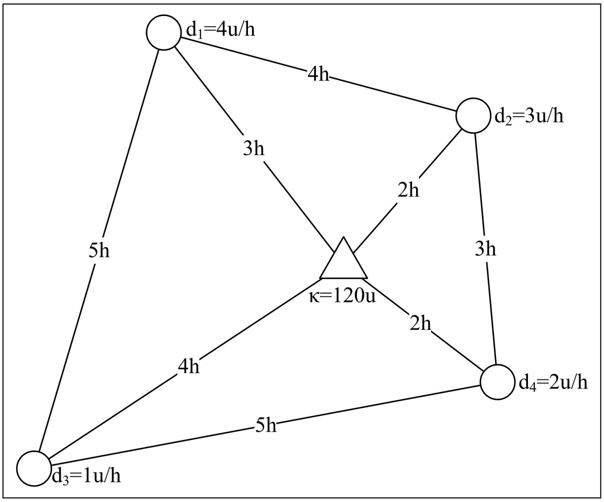


Figure 3.1: Illustrative example with 4 customers

First, the four basic distribution patterns are generated to find the customer priorities. These are: $\lambda_1 = 39$, $\lambda_2 = 33.42$, $\lambda_3 = 30.95$ and $\lambda_4 = 30.95$. The first customer has the highest priority and thus the initial solution consists of a single tour to this customer, with a cost rate of 39 euro per hour.

Then, customer 2, i.e. the one with the second highest priority, is inserted. The following alternatives are evaluated:

- A new distribution pattern is constructed with a single tour that visits customer 2.
- A new tour is constructed that visits customer 2, and this tour is added to the existing distribution pattern.
- Customer 2 is inserted into the existing tour of the existing distribution pattern.

The cheapest of these three alternatives is the second one, with a cost rate of 52.42 euro per hour. The current solution then exists of one distribution pattern consisting of two tours, one to each customer. This distribution pattern is in fact a multi-tour, since both tours have a frequency of 1.

Next, customer 3 is inserted. These are the possible alternatives.

- A new distribution pattern is constructed with a single tour that visits customer 3.
- A new tour is constructed that visits customer 3, and this tour is added to the existing distribution pattern.
- Customer 3 is inserted into the first tour of the existing distribution pattern.
- Customer 3 is inserted into the second tour of the existing distribution pattern.

Again, the cheapest alternative is the one where the customer has its separate tour in the existing distribution pattern. The current solution is then one distribution pattern with three tours and a cost rate of 63.58 euro per hour. In this distribution pattern, the tours to customers 1 and 2 have frequency 2, while the tour to customer 3 has frequency 1.

Finally, customer 4 is inserted. The following alternatives are considered.

- A new distribution pattern is constructed with a single tour that visits customer 4.
- A new tour is constructed that visits customer 4, and this tour is added to the existing distribution pattern.
- Customer 4 is inserted into the first tour of the existing distribution pattern.
- Customer 4 is inserted into the second tour of the existing distribution pattern.
- Customer 4 is inserted into the third tour of the existing distribution pattern.

The cheapest alternative is to insert customer 4 into the tour to customer 2. The frequencies of the tours to customer 1 and to customers 2 and 4 then become 3, while the frequency of the tour to customer 3 remains 1. The cost rate of this final solution is 74.24 euro per hour. This solution generated by the insertion heuristic is exactly the optimal solution that was already presented in the previous chapter (see p. 35). The different steps of the insertion heuristic are illustrated in Figure 3.2.

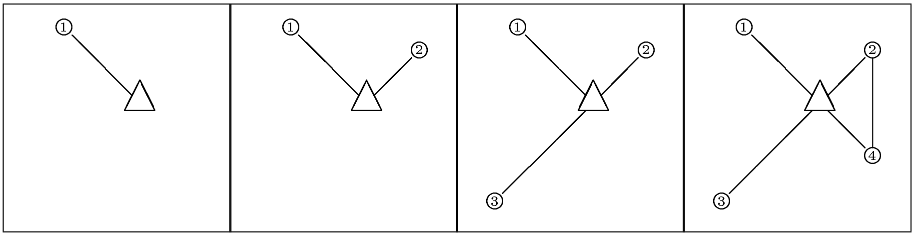


Figure 3.2: Applying the insertion heuristic to the illustrative example

3.2 A savings heuristic

The insertion heuristic above starts with an empty solution and then inserts customers one by one. In this section, an alternative construction heuristic is presented in which all customers are included in the solution from the beginning. The initial solution consists of a separate distribution pattern to each of the customers. Then, distribution patterns in the solution are combined pairwise in order to obtain a better solution. If two distribution patterns can feasibly be combined into one (i.e. a single vehicle can cover the work of two separate vehicles from the current solution) and the cost rate of this distribution pattern is smaller than the sum of the cost rates of the distribution patterns that are being combined, then a *saving* is realized through this combination. That is why this heuristic in which distribution patterns are being combined, is called a 'savings heuristic' [12].

The detailed steps of the savings heuristic are as follows.

1. A list L_d is initialized with the $|S|$ basic distribution patterns, each serving a single customer.

2. For each possible pair of distribution patterns D_i and D_j in L_d, a new distribution pattern D_{ij} is constructed that combines the two. If this results in a 'saving' (i.e. the cost rate of D_{ij} is smaller than the sum of the cost rates of D_i and D_j), this new distribution pattern is kept.

3. The distribution pattern combination D_{ij} that results in the largest saving is selected. The two constituent distribution patterns D_i and D_j are removed from L_d and replaced by D_{ij}.

4. Steps 2 and 3 are repeated as long as savings can be obtained by combining distribution patterns from L_d. The distribution patterns in L_d at the end of the procedure give the final solution.

In Step 2 of the savings heuristic, distribution patterns D_i and D_j are being combined. This combination is done according to the following procedure.

2.1 All tours from D_i and D_j are put together in a list L_t, and the distribution pattern D_{ij} making these tours is constructed.

2.2 For each possible pair of tours T_k and T_l from L_t, a new tour T_{kl} is constructed that merges them. Then, the distribution pattern D_{kl} is constructed that makes the list of tours L_{kl}, consisting of all tours from L_t, but with the single tour T_{kl} instead of the two tours T_k and T_l.

2.3 The cheapest distribution pattern D_{kl} from Step 2.2 is selected. If it is cheaper than the distribution pattern D_{ij} from Step 2.1, D_{ij} is overwritten by D_{kl}, and the list of tours L_t is overwritten by the list L_{kl}, after which the process returns to Step 2.2. Else, D_{ij} is returned as the distribution pattern combining D_i and D_j.

In Step 2.2, tours are being combined within distribution pattern D_0. It does not make sense here to try and combine two tours from D_i or two tours from D_j, because these combinations have already been evaluated before, when constructing D_i and D_j. Therefore, the set of possible tour combinations evaluated in Step 2.2 is restricted to the pairs T_k and T_l in which T_k is a tour from D_i and T_l is a tour from D_j.

The suggested savings heuristic is in fact a nested savings heuristic, because in the combination of distribution patterns , another savings heuristic is being used that combines tours within a distribution pattern.

Illustrative example

When applying the savings heuristic to our illustrative 4-customer example, this is what happens. The initial solution consists of 4 distribution patterns, one to each of the customers. Then, the following combinations are evaluated.

- The combination of the distribution pattern to customer 1 with the distribution pattern to customer 2.
- The combination of the distribution pattern to customer 1 with the distribution pattern to customer 3.
- The combination of the distribution pattern to customer 1 with the distribution pattern to customer 4.
- The combination of the distribution pattern to customer 2 with the distribution pattern to customer 3.
- The combination of the distribution pattern to customer 2 with the distribution pattern to customer 4.
- The combination of the distribution pattern to customer 3 with the distribution pattern to customer 4.

From these possibilities, the largest saving is obtained when combining the distribution patterns to customers 2 and 4. In this combination, two possibilities have been checked: the customers can be visited separately in two tours or can be visited together in a single tour. The latter, with a cost rate of 43.58 euro per hour, is the cheaper possibility. In the next step, the following combinations are evaluated.

- The combination of the distribution pattern to customer 1 with the distribution pattern to customers 2 and 4.
- The combination of the distribution pattern to customers 2 and 4 with the distribution pattern to customer 3.

The combination of the distribution pattern to customer 1 with the distribution pattern to customer 3 was already evaluated before. It does not have to be re-evaluated here, since the saving it realizes is still the same.

The combination with the largest saving is the second one. In the resulting distribution pattern, which has a cost rate of 54.54 euro per hour, there are two tours: the first tour, with frequency 3, goes to customers 2 and 4; the second tour, with a frequency of 1, covers customer 3. Combining these tours within this distribution pattern was considered, but it does not lead to a saving.

Finally, only the combination of the distribution pattern to customer 1 with the distribution pattern covering customers 2, 3 and 4 needs to be evaluated. In this combination, there are three possibilities: (i) customer 1 can remain in a separate tour, (ii) it can be inserted into the tour that already visits customers 2 and 4, and (iii) it can be inserted into the tour to customer 3. The first has a cost rate of 74.24 euro per hour and is the cheapest of these alternatives, such that the final solution consists of a single distribution pattern. In this distribution pattern, the tours to customer 1 and to customers 2 and 4 have frequency 3, while the tour to customer 3 has frequency 1. This resulting distribution pattern is again the optimal solution that was also obtained by the insertion heuristic. The different steps of the savings heuristic are shown in Figure 3.3.

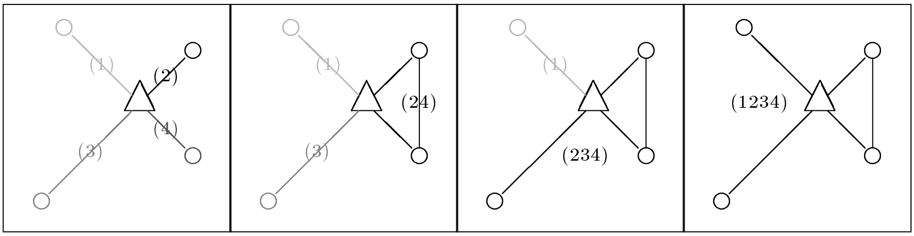


Figure 3.3: Applying the savings heuristic to the illustrative example

3.3 An improvement heuristic

In the previous sections, two constructive heuristics are presented that build solutions from scratch. In this section, an improvement heuristic is proposed that can be applied to an existing solution at any time to improve solution quality.

In this improvement heuristic, each customer is first removed from the solution and then re-inserted into it. Re-inserting a customer into the solution is very similar to inserting a customer into the solution in the insertion heuristic. The customer is inserted into the different distribution patterns of the solution, both in a separate tour and in the existing tours, and the cheapest alternative is kept. The alternative of putting the customer in a separate distribution pattern is not considered here. If the customer is re-inserted in the same position as where it was before removing it, the solution is restored and no improvement is found. If the customer ends up in a different position, it means that the solution has improved.

As in the insertion heuristic, the order in which the customers are considered for removal and re-insertion plays an important role. Therefore, the customers are considered according to the same priority, i.e. the cost rate of the corresponding basic distribution pattern. These are the different steps in our improvement heuristic.

1. Construct the basic distribution patterns to find the customer priorities λ_j. Sort the customers according to this priority.
2. Remove the customer with the next highest priority from the solution and then re-insert it. Consider inserting the customer into each of the existing distribution patterns. When inserting a customer into an existing distribution pattern, it can be inserted into one of the existing tours of the distribution pattern, or a separate tour to this customer can be added to the distribution pattern. The cheapest of all alternatives is kept.
3. Repeat the previous step for all customers in order of decreasing priority.
4. If the solution has changed, the process is restarted and all customers are removed and re-inserted once more.

When applying this improvement heuristic to the proposed solution for our illustrative example, all customers are re-inserted into the same position and no actual improvement is achieved.

This improvement heuristic and both construction heuristics are used to tackle the first two of five subproblems, i.e. (i) partitioning customers over a number of vehicles and (ii) sub-partitioning the customers of a vehicle over a number of tours. In evaluating each of the different alternatives that are encountered during these heuristics, the other three subproblems need to be solved: constructing minimal-length tours, determining tour frequencies and constructing delivery schedules. How these three issues are being dealt with in our solution approach is explained in the following three sections.

3.4 Constructing TSP-tours

When a subset of customers is assigned to a tour, the tour has to be actually constructed to be able to determine cost rates. The vehicle has to start from the depot, visit all of the customers in the subset in a certain sequence, and return to the depot. The sequence in which the customers are visited has to be such that the total transportation cost for the vehicle is minimal. This problem is known in literature as the Travelling Salesman Problem (TSP), and a wide variety of solution approaches are available for it. From all these approaches, a cheapest insertion heuristic is the obvious choice within our solution framework, since we are inserting customers into our solutions one by one, both in the constructive insertion heuristic and the improvement heuristic.

This cheapest insertion heuristic is as follows. When inserting a customer into a tour, all possible positions in the tour are examined: between the depot and the first customer, between the first and the second customer, ..., and finally, between the last customer and the depot. The customer is eventually inserted in the position for which the increase in transportation costs is minimal.

In our second construction heuristic, the savings heuristic, customers are not being inserted into tours one by one, but instead, tours are being combined. This is implemented by inserting the customers of the shorter of both tours in the longer of both tours one by one, using the cheapest insertion heuristic.

Customer time windows for delivery are not being considered. We assume that we are working under a VMI-strategy and the main idea there is that customers cooperate with the distributor, and therefore do not impose restrictive time windows. If, however, time windows would be considered, this would have its consequences for the tour construction. The straightforward cheapest insertion heuristic described here would then have to be adjusted to maintain time window feasibility.

3.5 Determining tour frequencies

Once the subset of customers assigned to a vehicle is partitioned into sub-subsets and tours are constructed for each sub-subset, the frequencies for these tours in the distribution pattern have to be determined. These frequencies should be such that the resulting cost rate (and thus also the resulting reduced cost rate) is minimal.

Every tour in a distribution pattern has its individual optimal cycle time (see Section 2.1). Thus, the frequencies should be chosen such that the resulting cycle times for the tours are as close as possible to their respective optimal cycle times. If the optimal cycle time of tour i is denoted T_i^*, then we are looking for a distribution pattern cycle time T and frequencies k_i such that $T/k_i \approx T_i^*$.

The smallest minimal cycle time that can be obtained for a distribution pattern is when all frequencies k_i are 1 (see Formula (2.25)). This is in fact the multi-tour solution. If the optimal cycle times of the tours are all smaller than this minimal cycle, the multi-tour gives the best result. Increasing frequencies would result in an increase of the distribution pattern cycle time and take the resulting tour cycle times further away from their optimal values, making the distribution pattern more expensive. For these cases, the tour frequency determining procedure introduced below need not be run.

If the optimal cycle times of the tours are not smaller than the cycle time of the multi-tour solution, it does make sense to have frequencies higher than 1. To determine the appropriate tour frequencies, we propose an iterative procedure that will evaluate a number of alternatives and keep the best one. In this procedure, we start from initial frequencies chosen such that there is at least one tour with frequency 1, for simplicity. The frequencies are then iteratively increased one by one until a stopping criterion is met. The obvious way to determine initial frequencies is the following: $k_i = \left\lfloor \frac{\max_k(T_k^*)}{T_i^*} \right\rfloor, \forall i$. The obvious stopping criterion is then to increase frequencies until $k_i \geq \left\lceil \frac{\max_k(T_k^*)}{T_i^*} \right\rceil, \forall i$.

As explained in Section 2.3, idle time is necessary in the schedule of a regular distribution pattern to make sure that deliveries are equidistant. For 'incompatible' frequencies (e.g. $k_1 = 2$ and $k_2 = 3$), this idle time can be very high, leading to a schedule time T_{sched} that is higher than the maximal cycle time T_{max}, which makes the distribution pattern infeasible. For distribution patterns with powers-of-two frequencies, much less idle time is needed, and the chance of having an infeasibility is much smaller. Therefore, we decided that the initial frequencies of the tours should be powers-of-two. Instead of the smallest integer smaller than $\frac{\max_k(T_k^*)}{T_i^*}$, the frequency is initialized with the smallest power-of-two smaller than $\frac{\max_k(T_k^*)}{T_i^*}$. The stopping rule also changes: instead of stopping at the next integer value, we now stop at the next power-of-two value.

1. The frequencies are initialized by rounding down to power-of-two values: $k_i = 2^{\left\lfloor \log_2\left(\frac{\max_k(T_k^*)}{T_i^*}\right) \right\rfloor}$. Cycle times and cost rates are calculated and a schedule is constructed for these initial frequencies.

2. The tour in the distribution pattern for which $k_i T_i^*$ is minimal, has its frequency increased by one. If this frequency is now more than double of the initial value ($k_i > 2^{1+\left\lfloor \log_2\left(\frac{\max_k(T_k^*)}{T_i^*}\right) \right\rfloor}$), the process stops. If not, changes in cycle times and cost rates are calculated.

3. A schedule is constructed. If it is feasible and the cost rate has decreased, these frequencies are kept. The process returns to Step 2.

The frequency determining procedure as described above is somewhat inefficient. In Step 2, a schedule is being constructed for every frequency combination that is encountered. This is not always necessary, because in some cases, we can already conclude that the considered frequency combination will not give a better result, without actually constructing the schedule. We have found four situations for which this is true, based on the following considerations.

- Increasing a frequency always leads to an increase of the minimal cycle time T_{min}, but not necessarily in an increase of the maximal cycle time T_{max}. The schedule time T_{sched} may even decrease, if the frequencies become more compatible. A useful measure of frequency compatibility (*FC*) is the least common multiple (lcm) of all frequencies k_i: $FC = \text{lcm}(k_i)$. If this least common multiple decreases by increasing one of the frequencies, it means that the frequencies have become more compatible. E.g. the combination $(1, 2, 3)$ is less compatible than $(1, 2, 4)$, because $\text{lcm}(1, 2, 3) = 6$ is bigger than $\text{lcm}(1, 2, 4) = 4$.

- We are looking for frequencies k_i and a cycle time T such that the resulting tour cycle times are close to the optimal tour cycle times: $T/k_i \approx T_i^*$. A useful measure of frequency optimality (*FO*) is the sum of the (absolute values of the) differences between T/k_i and T_i^*: $FO = \sum_i |T/k_i - T_i^*|$.

The four situations for which constructing the schedule can be skipped, are the following:

(I) If T_{min} has become bigger than T_{max}, the frequency combination is infeasible.

(II) If all k_i are even, the frequency combination gives the same result as the combination of which it is the double.

(III) If (i) the previous frequency combination was infeasible, (ii) T_{max} has not increased and (iii) the frequency compatibility measure *FC* has not decreased, then the current frequency combination will also be infeasible.

(IV) If the frequency optimality measure *FO* (with T the optimal cycle time in the interval $[T_{min}, T_{max}]$) has increased and the previous combination was feasible, then the current frequency combination cannot be better than the previous one.

The procedure to determine the right tour frequencies taking into account these considerations is then as follows.

1. The frequencies are initialized by rounding down to power-of-two values: $k_i = 2^{\lfloor \log_2(\frac{\max_k(T_k^*)}{T_i^*})\rfloor}$. Cycle times and cost rates are calculated and a schedule is constructed for these initial frequencies.

2. The tour in the distribution pattern for which $k_i T_i^*$ is minimal, has its frequency increased by one. If this frequency is now more than double of the initial value ($k_i > 2^{1+\lfloor \log_2(\frac{\max_k(T_k)}{T_i})\rfloor}$), the process stops. If not, changes in cycle times and cost rates are calculated. If none of the four situations (I), (II), (III) and (IV) described above occurs, the process goes to Step 3. Else, this step (2) is repeated without going to Step 3.

3. A schedule is constructed. If it is feasible and the cost rate has decreased, these frequencies are kept. The process returns to Step 2.

Illustrative example

To illustrate the frequency determining procedure, we apply it to our small 4-customer example that was introduced in the previous chapter. In the distribution pattern that has a separate tour for each of the customers, the tours have the following optimal cycle times: $T_1^* = 30$h, $T_2^* = 29.81$h, $T_3^* = 73.03$h and $T_4^* = 36.51$h. Table 3.1 shows the different iterations when determining the frequencies of these four tours.

Table 3.1: Determining the frequencies for the illustrative example

	k_i	*FC*	*FO*	(arg)$\min_i(k_i T_i^*)$	Schedule	Cost rate
1	$2,2,1,2$	2	19.73	59.63 (2)	yes	74.75
2	$2,3,1,2$	6	29.35	60.00 (1)	no (IV)	-
3	$3,3,1,2$	6	20.27	73.03 (3)	yes	74.68
4	$3,3,2,2$	6	36.71	73.03 (4)	no (IV)	-
5	$3,3,2,3$	6	34.73	89.44 (2)	yes	infeasible
6	$3,4,2,3$	12	41.85	90.00 (1)	no (III)	-
7	$4,4,2,3$	12	16.71	109.54 (4)	yes	infeasible
8	$4,4,2,4$	4	19.73	119.26 (2)	no (II)	-
9	$4,5,2,4$	20	25.35	120.00 (1)	no, STOP	

From iteration 5 on, the solutions are infeasible, because the schedule times become higher than the maximal cycle times. Iteration 8 is the double of the first iteration and is therefore skipped. In iteration 9, a frequency of 5 appears, which is more than the double of the initial frequency 2, so the process stops there.

3.6 Scheduling tours in a distribution pattern

Once tours are constructed and a set of frequencies k_i is determined, the distribution pattern needs to be scheduled. As explained in Section 2.3, the nature of this scheduling problem differs depending on two side-constraints: (i) equidistant deliveries and (ii) driving time restrictions.

Solving the scheduling subproblem is at the heart of our solution approach, since it is the fifth of the five nested subproblems in solving the cyclic inventory routing problem. To evaluate a large number of possible distribution patterns within a reasonable amount of time, the scheduling problem needs to be solved quickly. Solving the mathematical models presented in Section 2.3 to optimality is thus computationally unacceptable. Therefore, a fast heuristic is proposed that is adapted for the various versions of the problem.

3.6.1 Scheduling regular distribution patterns

In a regular schedule, the tour to S_i has to be made k_i times in a minimal period of time, such that the time between consecutive iterations of the same tour is constant. Below, an insertion heuristic is presented for scheduling the tours in a distribution pattern within a minimal time span, the so-called 'schedule time' T_{sched}. In this heuristic, tours are inserted one by one, and the schedule is 'stretched' if necessary (by increasing T_{sched}) to make sure that different tours do not have to be made at the same time. The parameters a_{il} and b_{il} are introduced to represent the time at which the l'th iteration of tour i starts in the cycle: $T_{il} = a_{il}T_{sched} + b_{il}$, $(i = 1..n, l = 1..k_i)$.

The suggested heuristic is a best-fit insertion heuristic. The order in which the different tours are added to the schedule is crucial for the performance of the heuristic. An obvious rule is to insert the tours in decreasing order of frequencies, and to take the longer tour first in case of a tie. However, as illustrated in an example below, this obvious rule is not very efficient. Therefore, the rule is extended such that no two tours with the same frequency are added to the schedule consecutively.

1. Initially, T_{sched} is set to T_{min} and the schedule is empty.
2. The tour i with the highest frequency (and with the longest travel time in case of a tie) is assigned to the start of the schedule, such that $T_{il} = \frac{l-1}{k_i}T_{sched}$ or $a_{il} = \frac{l-1}{k_i}$, $b_{il} = 0$ $(l=1..k_i)$.
3. If all tours have been added to the schedule, the procedure stops here and a feasible schedule is found. Else, determine the next tour to be added to the schedule. This is the tour with the highest frequency that is different from the frequency of the previously added tour (and with the longest travel time in case of a tie). The selected tour i has to be made k_i times within a cycle of T_{sched} hours, such that the first iteration has to be started at time $\frac{T_{sched}}{k_i} - TD_i$ at the latest.

4. For all gaps in the schedule before $\frac{T_{sched}}{k_i} - TD_i$.

   a. Write the timing t^* of the start of the gap as $aT_{sched} + b$.

   b. Schedule the first start of the selected tour i at t^*: $T_{i1} = aT_{sched} + b$, or $a_{i1} = a$, $b_{i1} = b$. The other iterations then start at: $T_{il} = \left(aT_{sched} + b + \frac{l-1}{k_i}T_{sched}\right) \bmod T_{sched}$, or $a_{il} = \left(a + \frac{l-1}{k_i}\right) \bmod 1$, $b_{il} = b$ $(l = 2..k_i)$.

   c. Check the schedule for overlaps. If there is no overlap, this timing for the newly added tour i is immediately accepted and the procedure returns to Step 3. If there is an overlap between the l'th iteration of the newly added tour i, which covers the interval $[a_{il}T_{sched} + b_{il}, a_{il}T_{sched} + b_{il} + TD_i]$ and the m'th iteration of a previously added tour j, covering interval $[a_{jm}T_{sched} + b_{jm}, a_{jm}T_{sched} + b_{jm} + TD_j]$, then T_{sched} has to be increased to make sure that one of these intervals is finished before the other begins.

      - if $a_{il} < a_{jm}$, we must have $a_{il}T_{sched} + b_{il} + TD_i \leq a_{jm}T_{sched} + b_{jm}$, or $T_{sched} \geq \frac{b_{il} + TD_i - b_{jm}}{a_{jm} - a_{il}}$.
      - if $a_{jm} < a_{il}$, we must have $a_{jm}T_{sched} + b_{jm} + TD_j \leq a_{il}T_{sched} + b_{il}$, or $T_{sched} \geq \frac{b_{jm} + TD_j - b_{il}}{a_{il} - a_{jm}}$.

      The t^* for which T_{sched} is minimal after the selected tour has been inserted and overlaps have been avoided, is eventually selected, making this a *best-fit* insertion heuristic.

5. If T_{sched} is not greater than the maximal cycle time T_{max}, return to Step 3. Else, the procedure stops here without finding a feasible solution.

In the illustrative example of Chapter 2, schedules of some proposed distribution patterns were given without further explanation. To illustrate the insertion heuristic suggested here, we show how the presented schedules are constructed.

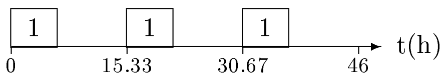
Illustrative example: the distribution pattern with adjusted frequencies

The 4-tour distribution pattern with frequencies $(3, 3, 1, 2)$ is a good example to illustrate the selection rule for the next tour to be inserted:

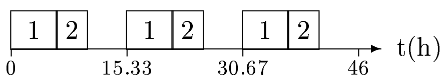
"the tour with the highest frequency that is different from the frequency of the previously added tour (and with the longest travel time in case of a tie)"

In this example, there are two tours with a frequency of 3, namely the tours to customers 1 and 2. First, the schedule is constructed assuming the obvious rule of always inserting the tour with the highest frequency.

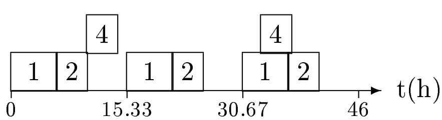
The initial schedule time is given by the minimal cycle time of 46 hours, and the first tour that is inserted to the schedule is the tour to customer 1 (because it is longer than the tour to customer 2). This tour then starts at times 0, $T_{sched}/3$ and $2T_{sched}/3$.



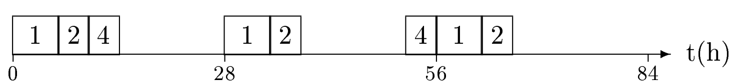
The second tour inserted into the schedule is the tour to customer 2, which also has frequency 3. The only gap in the schedule before $T_{sched}/3 - 4$ is found at time $t^* = 6$ hours. This gives the following start times for the second tour: 6, $\frac{T_{sched}}{3} + 6$ and $\frac{2T_{sched}}{3} + 6$. There are no overlaps, so T_{sched} does not have to be increased.



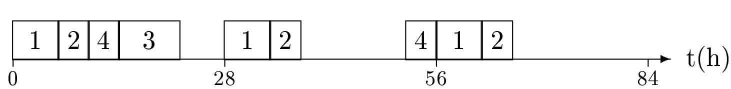
The third tour to be inserted is the tour to customer 4, with frequency 2. This tour can be inserted into two gaps: the gap at $t^* = 10$, and the gap at $t^* = \frac{T_{sched}}{3} + 10$. However, because of the symmetry, it will result in the same schedule, so the gap $t^* = 10$ is selected. The start times of the tour are then 10 and $\frac{T_{sched}}{2} + 10$.



However, the second iteration of this tour overlaps with the third iteration of the first tour, such that the schedule time has to be increased: $\frac{T_{sched}}{2} + 14 \leq \frac{2T_{sched}}{3}$, or $T_{sched} \geq 84$. The new schedule time is thus 84 hours. As this is not bigger than the maximal cycle time, the procedure continues.

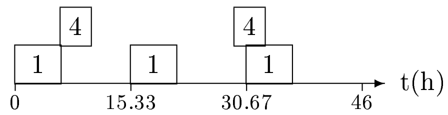


Finally, the tour to customer 3 is inserted. The first gap, at $t^* = 14$ is large enough to contain this tour, so that gives the starting time of the tour to customer 3. The schedule is then finished, having a makespan of 84 hours.

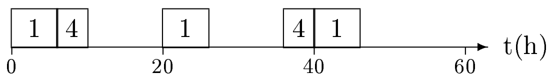


Now, the schedule is constructed with the actual selection rule, in which no two tours with the same frequency can be added consecutively.

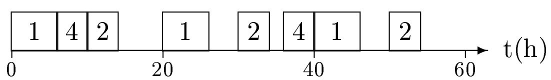
The initial schedule time is still 46 hours and the first tour that is inserted to the schedule is still the tour to customer 1, with starting times 0, $T_{sched}/3$ and $2T_{sched}/3$. However, the second tour to be inserted is now the tour to customer 4, with frequency 2. This tour can be inserted into two gaps: the gap at $t^* = 6$, and the gap at $t^* = \frac{T_{sched}}{3} + 6$. Because of the symmetry, this is the same, so the gap $t^* = 6$ is selected. The start times of the tour are then 6 and $\frac{T_{sched}}{2} + 6$.



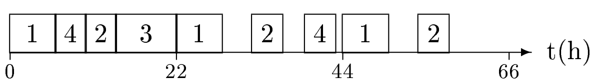
The second iteration of this tour overlaps with the third iteration of the first tour, such that the schedule time has to be increased: $\frac{T_{sched}}{2} + 10 \leq \frac{2T_{sched}}{3}$, or $T_{sched} \geq 60$. The new schedule time is thus 60 hours. This is not bigger than the maximal cycle time, so the procedure continues.



Next, the tour to customer 2 is inserted. The only available gap before $\frac{T_{sched}}{3} - 4$ is at $t^* = 10$, so the starting times of this tour are: 10, $\frac{T_{sched}}{3} + 10$ and $\frac{2T_{sched}}{3} + 10$.



Finally, the tour to customer 3 is inserted. The largest gap is found at $t^* = 14$, but it is not large enough to contain this tour, so that the schedule time has to be extended once more: $22 \leq \frac{T_{sched}}{3}$. This gives the final schedule time of 66 hours, which is much smaller than the schedule time of 84 hours found above.

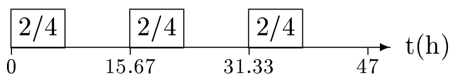


When inserting tours with the same frequency one after the other, they are ‘attached’ to each other in the schedule, such that they cover big blocks, making it harder to put tours with other frequencies in between. In our example, the first two tours cover three blocks of 10 hours, making it necessary to increase the schedule time a lot when inserting a tour with frequency 2. With the adjusted selection rule, this blocking is avoided by first inserting the tour with the lower frequency. The tours to customers 1 and 2 are then no longer attached to each other, but some free time is left in between them, such that the resulting schedule time is much smaller.

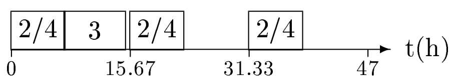
The optimal distribution pattern

The optimal solution for our illustrative example consists of a distribution pattern with three tours. The schedule is constructed as follows. The first tour to be inserted, is the tour to customers 2 and 4. It has the same frequency (3) as the tour to customer 1, but its travel time is longer (7 hours vs. 6).

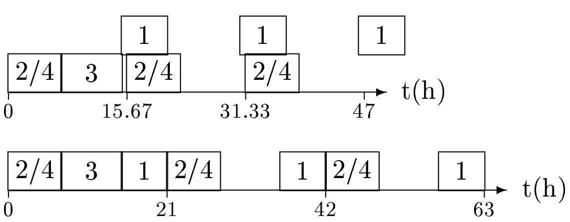
The schedule time is initially set to the minimal cycle time of 47 hours. The first tour is thus repeated every $47/3 = 15.67$ hours, at times 0, $T_{sched}/3$ and $2T_{sched}/3$.



Then, the tour to customer 3, is inserted because the tour to customer 1 also has frequency 3. This tour is started at time $t^* = 7$, giving no overlap.



Finally, the tour to customer 3 is inserted. The only available gap is the small one at time $t^* = 15$. The starting times of this tour are then 15, $T_{sched}/3 + 15$ and $2T_{sched}/3 + 15$. To avoid the overlaps, T_{sched} has to be increased such that $21 \leq \frac{T_{sched}}{3}$. The final schedule time is therefore 63 hours.



3.6.2 Regular scheduling under driving time restrictions

For the scheduling problem with driving time restrictions, a similar heuristic is proposed. Instead of stretching the schedule in a continuous way to avoid overlapping time intervals, the time horizon is now extended in discrete steps to avoid having to travel more than 8 hours per day.

Under driving time restrictions, cycle times are expressed as a number of days, and they have to be an integer multiple of the least common multiple of all frequencies k_i. This least common multiple gives the so-called base cycle time B. If driving during the weekend is not allowed, the base cycle time also has to be an integer multiple of 5. The proposed insertion heuristic for scheduling distribution patterns under driving time restrictions is as follows.

1. Initially, T_S is set to the minimal cycle time T_{MIN} and the schedule is empty.

2. The tour i with the highest frequency (and with the longest travel time in case of a tie) is assigned to the first day and thus has to be made on days $1 + (k-1)\frac{T_S}{k_i}$ (with $k = 1..k_i$).

3. If all tours have been added to the schedule, the procedure stops here and a feasible schedule is found. Else, determine the next tour to be added to the schedule. This is the tour with the highest frequency that is different from the frequency of the previously added tour (and with the longest travel time in case of a tie). The selected tour i has to be made k_i times within a cycle of T_S days, such that the first iteration has to be on a day in the interval $[1, \frac{T_S}{k_i}]$.

4. For t^* from 1 to $\frac{T_S}{k_i}$.

   a. The days on which tour i is made are: $t^* + (k-1)\frac{T_S}{k_i}$ (with $k = 1..k_i$). If the 8-hour restriction is violated on any of these days, go to the next t^*.

   b. For each of the days $t^* + (k-1)\frac{T_S}{k_i}$ (with $k = 1..k_i$), sum the time remaining on these days after inserting tour i. The t^* for which this cumulative remaining time is minimal, will eventually be selected, making this a *best-fit* insertion heuristic. By doing this, bigger blocks of idle time are left for inserting the remaining tours.

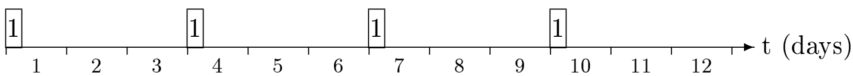
5. If a t^* is found that results in a feasible schedule, return to Step 3. Else, increase T_S by the base cycle time B. If T_S is not greater than the maximal cycle time T_{MAX}, return to Step 2. Else, the procedure stops here without finding a feasible solution.

Best-fit insertion

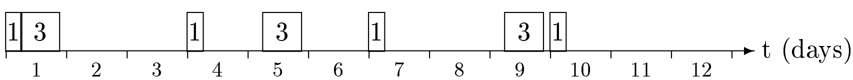
This section presents an example that illustrates the *best-fit* rule of Step 4b in the scheduling procedure, by considering a vehicle that has to perform the following 5 tours.

i	k_i	TD_i
1	4	2h
2	3	3h
3	3	5h
4	2	6h
5	1	7h

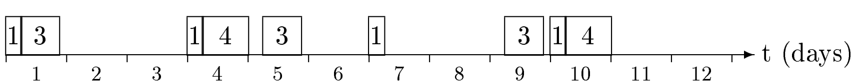
The base cycle time is 12 days, and the total driving time is 51 hours, so the initial schedule has a time horizon of 12 days and tour 1 is made on days $1, 4, 7$ and 10.



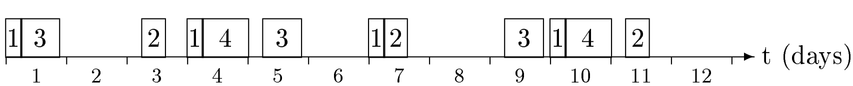
The second tour to be inserted is tour 3: it has the highest frequency that is different from 4 and it is longer than tour 2, which also has a frequency of 3. When assigning it to day $t^* = 1$, the tour ends up in days $1, 5$ and 9. The first tour is also made on day 1, but together they only take 7 hours, so the driving time restriction is not violated. The remaining times are 1 hour on day 1, and 3 hours on days 5 and 9. The cumulative remaining time is thus 7 hours. Assigning the tour to $t^* = 2, 3$ and 4 is also feasible, but does not give a lower cumulative remaining time. The selected assignment for tour 3 is therefore $t^* = 1$.



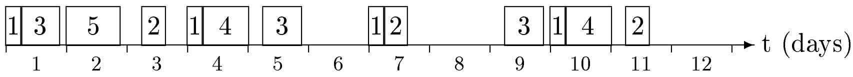
Then, tour 4 is inserted. Tour 2 has a higher frequency, but this frequency is equal to the frequency of the previously added tour 3. With $t^* = 1$, there is an overflow on day 1, with $t^* = 3$, there is an overflow on day 9, and with $t^* = 5$, there is an overflow on day 5. With $t^* = 2$ or $t^* = 6$, tour 4 is made on days on which no other tour is yet scheduled, giving a cumulative remaining time of 4 hours. With $t^* = 4$, tour 4 is put on days 4 and 10, together with tour 1, such that the cumulative remaining time is 0 hours, such that $t^* = 4$ is selected.



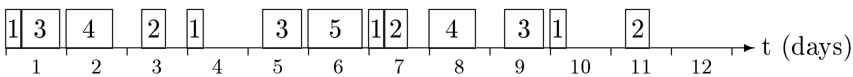
Next, tour 2 is inserted. Days $t^* = 1$, $t^* = 2$ and $t^* = 4$ are infeasible, due to driving time restriction violations on respectively day 1, 10 and 4. The only feasible assignment is $t^* = 3$. This puts tour 2 on days $3, 7$ and 11.



Finally, tour 5 is inserted. This tour takes 7 hours and can not be combined with any other tour. The tour is therefore assigned to $t^* = 2$, the first empty day.

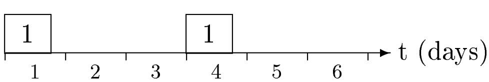


If the procedure would be a first-fit heuristic instead of a best-fit heuristic, the schedule for this example would have looked quite different (see below). The assignment of tour 4 would have been to $t^* = 2$, and tour 5 would be made on day $t^* = 6$. It is obvious that the former schedule is more interesting from a practical point of view, because it has three completely free days for the vehicle instead of only one. On these free days, the vehicle can be used for other purposes than executing the cyclic replenishment scheme.

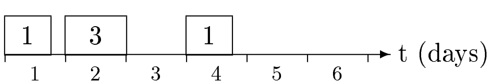


Illustrative example: the powers-of-two distribution pattern

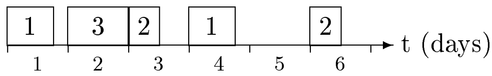
For our illustrative 4-customer example, two feasible solutions are presented when the driving time is restricted to 8 hours per day. The first of these solutions is the so-called powers-of-two solution. This distribution pattern has frequencies $(2, 2, 1, 2)$, so the base cycle time is 2 days. The total travel time is 36 hours or 4.5 days, so the initial schedule time is set to 6 days, which is the smallest multiple of the base cycle time larger than 4.5. The first tour to be inserted is the one to customer 1. With $t^* = 1$, this tour ends up in days 1 and 4.



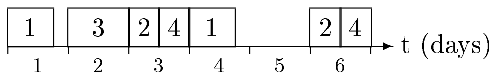
Next, the tour to customer 3 is inserted, because the tours to customers 2 and 4 have the same frequency as the previously inserted tour. With $t^* = 1$, the driving time restriction would be violated, so $t^* = 2$ is selected.



Then, the tour to customer 2 is inserted. With $t^* = 1$ and $t^* = 2$, the driving time restriction would be violated, so $t^* = 3$ is selected. This puts the tour to customer 2 in days 3 and 6.



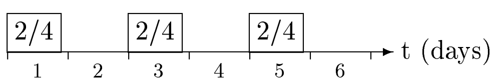
Finally, the tour to customer 4 is inserted. The only feasible assignment is $t^* = 3$, which puts the tours to customers 2 and 4 in the same day. This is feasible, because together, both tours take exactly 8 hours.



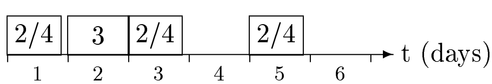
The optimal distribution pattern

To illustrate the regular scheduling heuristic under driving time restrictions, it is applied to the optimal solution of the illustrative 4-customer example of Chapter 2.

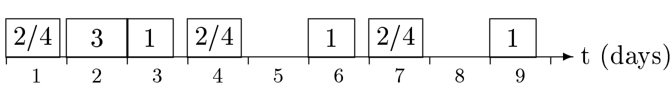
With frequencies $(3, 3, 1)$, the base cycle time is 3 days. However, because the total travel time is 47 hours or 5.875 days, the initial schedule time is set to 6 days. The first tour that is inserted is the one to customers 2 and 4. With $t^* = 1$, this tour ends up in days 1, 3 and 5.



Next, the tour to customer 3 is inserted. With $t^* = 1$, the driving time restriction would be violated, so $t^* = 2$ is selected.



Finally, the tour to customer 1 has to be inserted. However, both $t^* = 1$ and $t^* = 2$ are infeasible. Therefore, the schedule time is increased by $B = 3$ days to 9 days. This brings the tour to customers 2 and 4 on days 1, 4 and 7 while the tour to customer 3 remains on day 2. Now, the last tour can be inserted on day $t^* = 3$ without violating driving time restrictions.



3.6.3 Scheduling irregular distribution patterns

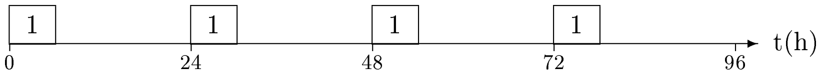
Although irregular distribution patterns are not actually being considered in our computational experiments, we still discuss the irregular distribution pattern scheduling problem shortly. This scheduling problem may look completely different from regular scheduling, but there is in fact a lot of similarity. In the irregular case, holding costs need to be minimized within a given cycle time T. This can be obtained by spreading different deliveries to the same customer as good as possible. In the regular case, a perfect spreading of the different deliveries is imposed and the cycle time T_{sched} is a variable. Thus, the regular schedule that is found is the ideal schedule for the irregular case, but it is infeasible if T_{sched} is larger than the (approximated) optimal cycle time T, which is the makespan of the irregular schedule.

In regular scheduling, we start with the schedule time T_{sched} equal to the minimal cycle time T_{min} and then increase it to avoid overlaps of time intervals that need to be equidistant. In irregular scheduling, we just work the other way around: the makespan of the schedule is fixed, and to avoid overlapping intervals, the intervals themselves are shifted forward or backward.

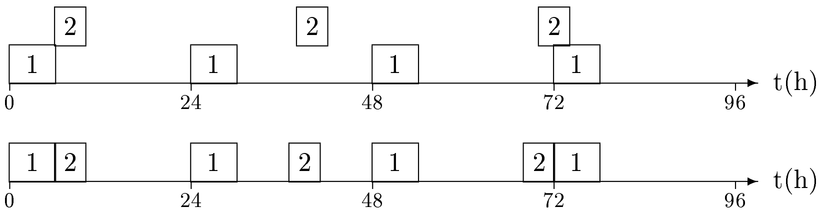
Illustrative example: the full-truckload distribution pattern

In our illustrative example, the so-called 'full-truckload' distribution pattern has an optimal cycle time of 93 hours. However, the minimal schedule time when deliveries have to be equidistant is 120 hours. Thus, a cycle time of 93 hours can only be achieved if an irregular distribution pattern is allowed. Here, we construct the irregular schedule when the cycle time is rounded to 96 hours.

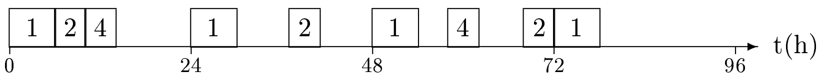
The tour to customer 1 has the highest frequency and is therefore inserted first. It is started every 24 hours, at times 0, 24, 48 and 72.



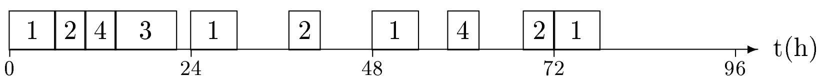
The tour to customer 2 has frequency 3 and is inserted next. Due to symmetry, both possible gaps give the same result and thus the gap at $t^* = 6$ is chosen. This results in the following starting times for the second tour: 6, 38 and 70. The last iteration has an overlap of 2 hours with the last delivery to customer 1. To avoid this overlap, the deliveries to customer 2 are shifted in time a bit. The last delivery to customer 2 is started 2 hours earlier, at 68 hours and the second delivery to customer 2 is started 1 hour earlier, at 37 hours.



Next, the tour to customer 4 is added. There are three possible gaps: at $t^* = 10$, $t^* = 30$ and $t^* = 42$. Inserting into the first gap gives starting times 10 and 58, which gives no overlap, so this is immediately selected.

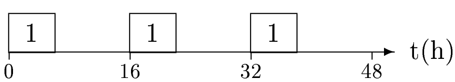


Finally, the tour to customer 3 is inserted. Again, the first gap, at $t^* = 14$, does not cause problems and the starting time of this tour is thus at 14 hours. The irregular schedule is then completed, and only the deliveries to customer 2 had to be slightly shifted.

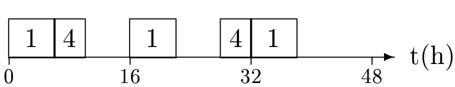


The 4-tour distribution pattern with adjusted frequencies

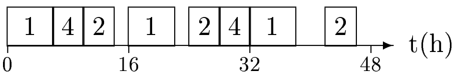
In an alternative solution for our illustrative example, the customers are still in separate tours, but the frequencies are adjusted to $(3, 3, 1, 2)$. Although a regular schedule can be found for the optimal cycle time of 84 hours, an irregular schedule for a cycle time of 48 hours is constructed here to illustrate the irregular scheduling heuristic. The tour to customer 1 is inserted first, at times 0, 16 and 32.



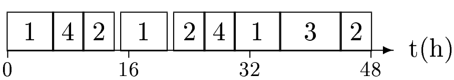
Then, the tour to customer 4 is inserted in the first gap, at $t^* = 6$. This gives starting times 6 and 30. However, this second iteration overlaps with the third iteration of the first tour, and is therefore started 2 hours earlier, after 28 hours.



The next tour to be inserted is the tour to customer 2, with a frequency of 3. It can only be assigned to the first gap, at $t^* = 10$. The resulting starting times are 10, 26 and 42. This second delivery overlaps with the second delivery to customer 4 and is therefore shifted 2 hours forward. The other iterations of this tour are not shifted forward or backward, because this only increases the deviation from having equidistant deliveries.



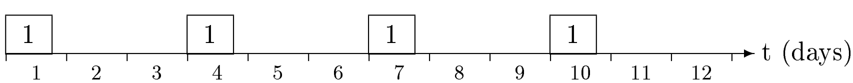
Finally, the tour to customer 3 is inserted. This tour takes 8 hours, while the largest gap in this schedule is only 4 hours, at $t^* = 38$. To make a feasible schedule, the deliveries before and after this largest gap are shifted: the second delivery to customer 2, the second delivery to customer 4 and the third delivery to customer 3 are started 2 hours earlier and the third delivery to customer 2 is started 2 hours later. To compensate the earlier start of the third iterations of the tour to customer 1, its second iteration is started 1 hour earlier. In this schedule, the only customer receiving equidistant deliveries is customer 3, but this customer has equidistant deliveries by definition since it is only served once per cycle. This schedule is highly irregular because (i) frequencies 2 and 3 are incompatible, and (ii) only 2 hours of idle time are available.



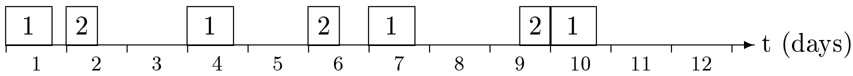
The full-truckload distribution pattern with driving time constraints

When driving time restrictions apply, the 'full-truckload' distribution pattern with a cycle time of 96 hours or 12 days is feasible. The construction of its schedule is illustrated here.

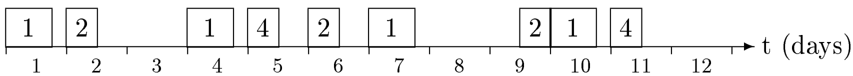
The tour to customer 1 has the highest frequency and is inserted first. The time between deliveries should be 3 days and therefore this tour is assigned to days 1, 4, 7 and 10.



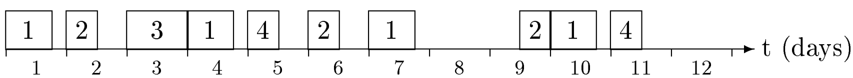
The next tour to be inserted is the one visiting customer 2. It has frequency 3 and is to be repeated every 4 days. Since it is impossible to achieve inter-delivery times of exactly 4 days due to the presence of the first tour, the following happens. When assigning the tour to day $t^* = 2$, the third and last iteration overlaps with the last iteration of the first tour. This third and last iteration is therefore shifted forward to the last 4 hours of the previous day. To compensate this, it is tried to shift the second iteration of this tour forward with two hours. However, this is infeasible since this second iteration is started at the beginning of day 6.



Next, the tour to customer 4 is inserted. It has frequency 2 and should thus be repeated every 6 days. Assigning this tour to days 5 and 11 gives exactly this.



Finally, the tour to customer 3, which takes a whole 8-hour day, is assigned to the first free day, i.e. day 3. As for the case without driving time restrictions, it is only customer 2 who no longer receives equidistant deliveries.



3.7 The multi-start framework

The insertion and savings heuristics, combined with the improvement heuristic, are fast heuristics that generate a single solution when applied to a problem instance. Since we are considering a long-term problem and not a real-time problem, the objective is to come up with good solutions for the problem at hand within a reasonable amount of time, instead of finding an acceptable solution as fast as possible. Theoretically, the proposed cyclic solutions will be used over an infinite time horizon. In practice, this will mean that the solutions are re-evaluated every few months. Therefore, we assume that computation times are not restricted to seconds or minutes, but to hours or even days. Because the proposed heuristics generate a solution very quickly, they can be reused in an attempt to try and find even better solutions.

In a multi-start solution framework, a heuristic is applied a number of times until a stopping criterion is met. In the different iterations, different solutions are generated and the best solution is kept. Possible stopping criteria are: (i) a fixed number of iterations, (ii) a limited amount of time, (iii) a number of iterations without finding a new best solution. The stopping criterion that we have adopted was the first, with the number of iterations equal to the number of customers in the problem instance.

When using the insertion and savings heuristics exactly as they are presented above in a multi-start framework, the same solution would be found in each iteration. Therefore, the heuristics have to be slightly adapted.

For the insertion heuristic, the order in which the customers are inserted into the solution should be different in each iteration. This can be achieved by assigning different priorities to the customers in each iteration. We decided to use a simple cost allocation system for this priority. The cost rate of the distribution patterns in a solution from one iteration is allocated to the customers to give priorities for the next iteration. Expensive customers thus have a higher priority in the next iteration. The allocation of the cost rate of a distribution pattern to the customers it serves, is as follows.

- First, the fixed vehicle cost rate is divided over the different tours of the distribution pattern proportional to the time required for these tours:

$$\psi_{i'} = \psi \frac{k_{i'}\left(t_\Delta + T_{TSP(S_{i'}+\Delta)} + \sum_{j\in S_{i'}} t_j\right)}{\sum_{i=1}^{n} k_i\left(t_\Delta + T_{TSP(S_i+\Delta)} + \sum_{j\in S_i} t_j\right)}$$

- Next, the fixed vehicle cost rate that is assigned to tour i is further divided over the customers in that tour proportional to their demand rates:

$$\psi_{j'} = \psi_i \frac{d_{j'}}{\sum_{j\in S_i} d_j}$$

- Then, the dispatching and transportation costs of tour i are divided over the customers in that tour, also proportional to their demand rates:

$$\chi_{j'} = \left(\varphi_\Delta + C_{TSP(S_i+\Delta)}\right) \frac{d_{j'}}{\sum_{j\in S_i} d_j}$$

- Finally, the priority assigned to a customer consists of the divided fixed vehicle costs plus the divided fixed tour costs plus the customer's individual delivery and holding cost rate:

$$\lambda_j = \psi_j + \frac{k_i}{T}\left(\chi_j + \varphi_j\right) + \frac{T}{k_i}\left(\frac{\eta_j d_j}{2}\right)$$

If in a certain iteration a solution is found that has been found in a previous iteration, the cost allocation obviously gives the same priorities as in that previous iteration. This means that the solution found in the next iteration will also be a solution that has been found before and the same cycle of solutions will start re-appearing. To avoid this cycling, all solutions are kept and if a solution is encountered that already exists, the customer priorities are randomized before starting the next iteration. This randomization consists of multiplying the priorities obtained from the cost allocation by a uniformly generated number between 0 and 10.

In the savings heuristic, the saving resulting from the combination of two distribution patterns does not depend on the customer priorities. To find different solutions in the different iterations of the savings heuristic in a multi-start framework, *reduced cost rates* are introduced. The reduced cost rate of a distribution pattern is the cost rate of that distribution pattern minus the sum of the priorities of all customers that it covers. The saving of combining two distribution patterns is then calculated as the sum of the reduced cost rates of the two constituent distribution patterns minus the reduced cost rate of the combined distribution pattern. By using reduced cost rates, the saving of a distribution pattern combination does depend on the customer priorities, such that the order in which combinations are selected in the savings heuristic may vary in different iterations.

However, the savings heuristic turns out to be rather robust with regard to customer priorities, i.e. the same final solution is often found for varying customer priorities. To avoid always ending up in one of only a limited set of alternative solutions due to this robustness, a second adaptation was made to the savings heuristic. For this, a distribution pattern also gets a priority assigned to it. This priority is the sum of the priorities of the customers it covers. Then, instead of evaluating all possible distribution pattern combinations and selecting the one with the largest saving, only the combinations with the distribution pattern with highest priority are considered. If there is no feasible combination with this distribution pattern, combinations with the distribution pattern with second highest priority are considered, and so on. By thus restricting the possible distribution pattern combinations, the heuristic is much more dependent on the customer priorities and more different solutions are obtained in the different iterations. Again, randomized priorities are used if necessary to avoid cycling.

The multi-start procedure using the adjusted heuristics, then has the following steps.

1. Generate the 'basic' distribution patterns, each serving a single customer $j \in S$ and use their cost rates for the customer priorities λ_j. Sort the customers according to this priority. Keep the set of basic distribution patterns as the current best solution.

2. Repeat the following steps (3 to 6) $|S|$ times.

3. Perform the adjusted savings heuristic, using reduced cost rates, to construct a solution and apply the improvement heuristic to it. If this solution is better than the current best solution, overwrite the current best solution with this one.

4. Generate new customer priorities from this solution and randomize them if necessary.

5. Perform the insertion heuristic to construct a solution and apply the improvement heuristic to it. If this solution is better than the current best solution, overwrite the current best solution with this one.

6. Generate new customer priorities from this solution and randomize them if necessary.

3.8 The column generation framework

A distribution pattern is a single vehicle replenishing a subset of customers at a certain cost rate. What we need to find is a collection of distribution patterns such that (i) every customer is replenished by one of the vehicles and (ii) the sum of the cost rates of these distribution patterns is minimal.

In other words, a partition of the set of customers S into disjoint subsets, each covered by a separate distribution pattern, needs to be found, such that the sum of cost rates of the distribution patterns is minimal. If we let DP denote the collection of all distribution patterns, this problem is formulated as follows.

$$\text{(MP) Minimize } TCR = \sum_{d \in DP} C^d X^d$$

subject to:

$$\sum_{d \in DP} A_j^d X^d = 1 \quad \forall j \in S \tag{3.1}$$

$$X^d \in \{0,1\} \qquad \forall d \in DP$$

where C^d is the cost rate of distribution pattern $d \in DP$, and A_j^d is the binary matrix stating whether distribution pattern $d \in DP$ covers customer $j \in S$ or not. The binary variable X^d indicates whether distribution pattern $d \in DP$ is selected or not.

The set DP of possible distribution patterns, i.e. the number of columns in problem (MP), is infinitely large. Therefore, we need an efficient way of producing only the most promising distribution patterns in DP, such that a (near-)optimal solution for the problem is quickly reached. This consideration made us decide on adopting a column generation framework. The column generation procedure consists of the following steps.

1. The set of distribution patterns DP is initiated with the $|S|$ 'basic' distribution patterns, i.e. those that visit only one customer. A separate distribution pattern is thus constructed for each customer, and the initial matrix A is the identity matrix.

2. The linear programming relaxation of the 'restricted' master problem (MP) is solved. The relaxation consists in the fact that the X^d variables are allowed to take fractional values, while the restriction is in the fact that DP is only a subset of all possible distribution patterns.

3. The LP-relaxed solution gives us a dual price for each of the customers, $\lambda_j, \forall j \in S$. These dual prices are obtained from Constraints (3.1).

4. The dual prices of the customers are used in an attempt to construct new distribution patterns with negative reduced cost rates. This is the column generation subproblem.

   (a) Use the dual prices λ_j as customer priorities.

   (b) Perform the adjusted savings heuristic, using reduced cost rates, to construct a solution and apply the improvement heuristic to it.

   (c) Perform the insertion heuristic to construct a solution and apply the improvement heuristic to it.

   If new distribution patterns with negative reduced cost rates are found in steps (b) and (c), they are added to the set *DP* and the corresponding columns are added to the matrix A. The process then returns to Step 2. If no such distribution patterns are found, we move to Step 5.

5. The restricted master problem is solved to integrality ($X^d \in \{0,1\}$) to provide the final solution. Remember that *DP* is now the set of all distribution patterns generated during the column generation process.

Column generation

Column generation is a strategy to extend a linear program columnwise as needed in the solution [24]. When the subproblem of finding the least reduced cost column is solved optimally, column generation is an exact solution approach for linear programs.

When applied to integer linear programs, column generation does not guarantee an optimal solution, but still it is often used for the following two purposes.

- The optimal solution of the LP-relaxed master problem can be used as a lower (or upper) bound for the integer optimum. This bound can be used to evaluate heuristic solution approaches.

- Column generation can be combined with branch-and-bound in optimal, so-called *branch-and-price* procedures [7].

In our application, the master problem is an integer linear program. However, since the subproblem is not solved to optimality, our column generation serves neither of the above purposes. As such, our column generation procedure is a mathematical programming based heuristic with no guarantee of optimality. In fact, there is much similarity between our column generation framework and our multi-start framework. In both approaches, the insertion and savings heuristic are iterated a number of times with different customer priorities such that a wide variety of good solutions is obtained. The difference between both approaches is two-fold.

1. Customer priorities.
   In the multi-start framework, customer priorities are generated by distributing the cost rates of the distribution patterns in the last proposed solution over the customers in a cost allocation system. In the column generation framework, customer priorities are obtained from the dual prices of the customers in the master problem. These dual prices hold information on all previously generated solutions that were added as columns in the master problem, instead of just the information from the last proposed solution.

2. The final solution.
   In the multi-start framework, the final solution is just the cheapest of all solution proposals encountered in the different iterations. In the column generation framework, all solutions are kept as columns in the master problem, and mathematical programming is used to select an appropriate subset of distribution patterns. This may correspond exactly to one of the solution proposals suggested by the savings or insertion heuristic in one of the iterations, but it may also consist of distribution patterns from different solutions proposals that constitute an even better solution together.

Tailoring the column generation procedure

In traditional column generation, e.g. when applied to the cutting stock problem, the subproblem is solved to optimality and only a single column with a negative reduced cost is added to the restricted master problem per iteration. In our case, adding a single column per iteration would cause some problems.

The constraints from which the dual prices are obtained, are equality constraints. This means that the dual prices λ_j can be both positive and negative. Further, the number of distribution patterns needed to visit all customers is always less than the number of customers. In other words, the number of columns (distribution patterns) used for a solution of the restricted master problem is less than the number of rows (or constraints). This means that solutions are always degenerate, and from the same solution, different shadow prices can be derived, depending on which columns are used to fill the basis.

When only a single new distribution pattern (or column) with a negative reduced cost rate is added per iteration, this new column will certainly become part of the basis. Shadow prices then change, but when implementing this, it turns out that only one shadow price changes, namely the shadow price of the customer that had the highest shadow price before. As a result, the distribution pattern constructed in the next iteration has a high chance of visiting the same customers, except for that one customer. This way, for large problem instances, it takes a long time before new distribution patterns are generated for customers which have smaller shadow prices.

An even more important problem is encountered in Step 5. Because of the overlap of the different distribution patterns that are being generated, it is difficult to find a good integer solution, i.e. a partitioning of the set of customers. When only a single distribution pattern is generated, usually it visits one or more customers that the previous distribution pattern (column) already visits. Consider e.g. a problem instance with 6 customers. Suppose that in each of the subproblems, a distribution pattern visiting four of these customers can be constructed. These four-customer distribution patterns are all overlapping, so only one of them can be selected for the final integer solution, meaning that two of the basic distribution patterns are required in the final solution. The actual optimal solution most probably consists of only two distribution patterns visiting three customers, or maybe one visiting two and the other one visiting four customers. The problem is that these two- and three-customer distribution patterns are not added as columns in the restricted master problem, even though they may be encountered at a certain moment as intermediate solutions in the distribution pattern generation subproblem.

To avoid these problems, we decided to construct complete solutions in the subproblem and to add all intermediate distribution patterns with a negative reduced cost that are encountered as new columns in the restricted master problem. Furthermore, since the solution proposal in a single iteration may be better than any solution encountered so far, all distribution patterns in that solution proposal are added to the restricted master problem, regardless of their reduced cost rates.

In the column generation subproblem, heuristics are used. It may happen that these heuristics do not find columns with negative reduced costs, even though they exist. If this is the case, the shadow prices will remain the same in the restricted master problem and the column generation procedure will be stopped prematurely. To prevent this from happening, the same extension as in the multi-start procedure is used: randomization of the priorities. When the shadow prices remain unchanged, instead of stopping with generating columns, the shadow prices are multiplied with a uniformly generated random number between 0 and 10, and the column generation is continued. However, if randomization would always be performed after shadow prices remain unchanged, the column generation would run infinitely. Therefore, the number of shadow price randomizations is limited to 5, so when the shadow prices remain the same for the sixth time, the generation of columns is stopped after all. The outline of the column generation is then as follows.

1. The set of distribution patterns *DP* is initiated with the $|S|$ 'basic' distribution patterns, i.e. those that visit only one customer. A separate distribution pattern is thus constructed for each customer, and the initial matrix A is the identity matrix.

2. The linear programming relaxation of the restricted master problem is solved.

3. Constraints (3.1) give us a dual price for each of the customers, $\lambda_j, \forall j \in S$.
4. If the dual prices of the customers are the same as in the previous iteration, they are randomized by multiplying them with a uniformly generated random number between 0 and 10. If this is the sixth time that the shadow prices are randomized, the process goes to Step 5. Else, generate new columns.
   (a) Use the dual prices λ_j as customer priorities.
   (b) Perform the adjusted savings heuristic, using reduced cost rates, to construct a solution and apply the improvement heuristic to it.
   (c) Perform the insertion heuristic to construct a solution and apply the improvement heuristic to it.

   If new distribution patterns with negative reduced cost rates are found in steps (b) and (c), they are added to the set *DP* and the corresponding columns are added to the matrix A. Furthermore, if a new best solution is encountered, all distribution patterns with positive reduced cost rates are also added to the set *DP* and their corresponding columns added to the matrix A. The process then returns to Step 2.
5. The restricted master problem is solved to integrality ($X^d \in \{0, 1\}$) to provide the final solution. Remember that *DP* is now the set of all distribution patterns generated during the column generation process.

In Step 4 of the column generation process, the so-called subproblem, new distribution patterns are constructed and added to the master problem. However, when executing the insertion and savings heuristic, it can happen that a 'new' distribution pattern is encountered that covers the same set of customers as another distribution pattern that is already included in *DP*, but for which the organisation of tours is different. In this case, only the cheaper of both is kept in *DP*.

Mixed integer model for the subproblem

In Table 2.3 of Section 2.2, a mixed integer formulation is presented for the cyclic inventory routing problem, when using the routing concept of multi-tours. In this paragraph, a mixed integer formulation is given for the subproblem in a column generation approach, also when restricted to the multi-tours. The notations used in this formulation are the following.

- S^+ is the set of 'locations', indexed by j, k and l. It consists of the set of customers S and the depot Δ.
- c_{jk} is the variable transportation cost between locations j and k.
- t_{jk} is the travel time between locations j and k.

The following variables are used in the model of the multi-tour generation subproblem, shown in Table 3.2.

- T is the cycle time of the multi-tour that is being constructed.
- X_{jk} is a binary variable that indicates whether the vehicle travels from location $j \in S^+$ to location $k \in S^+$ in this multi-tour or not.
- Z_{jk} is the cumulative demand rate of all remaining customers in a tour when the vehicle goes from location $j \in S^+$ to location $k \in S^+$. It is zero if the vehicle does not go directly from j to k. This additional variable is needed to impose that tours start and end in the depot.

Table 3.2: Mixed integer model for the multi-tour generation subproblem

$$\text{Min RCR} = \psi + \sum_{j \in S^+} \sum_{k \in S^+} X_{jk} \left(\frac{1}{T}(c_{jk} + \varphi_j) + T\frac{\eta_j d_j}{2} \right) - \sum_{j \in S} \sum_{k \in S^+} \lambda_k X_{jk}$$

subject to:

$$\sum_{j \in S^+} X_{jk} \leq 1 \qquad \forall k \in S \tag{3.2}$$

$$\sum_{j \in S^+} X_{jk} = \sum_{l \in S^+} X_{kl} \qquad \forall k \in S^+ \tag{3.3}$$

$$\sum_{j \in S^+} \sum_{k \in S^+} X_{jk}(t_{jk} + \varphi_j) \leq T \tag{3.4}$$

$$\sum_{k \in S^+} X_{jk} \leq f_j T \qquad \forall j \in S \tag{3.5}$$

$$\sum_{j \in S^+} Z_{jk} = d_k + \sum_{l \in S^+} Z_{kl} \qquad \forall k \in S \tag{3.6}$$

$$Z_{jk} \leq \left(\sum_{l \in S} d_l \right) X_{jk} \qquad \forall j, k \in S \tag{3.7}$$

$$T Z_{\Delta j} \leq \kappa \qquad \forall j \in S \tag{3.8}$$

$$T d_j \leq \kappa_j \qquad \forall j \in S \tag{3.9}$$

$$T, Z_{jk} \geq 0, X_{jk} \in \{0, 1\} \qquad \forall j, k \in S^+$$

The first constraint, (3.2), imposes that a customer is visited at most once. Constraint (3.3) is the vehicle flow conservation constraint: the number of vehicles entering a location is equal to the number of vehicles leaving that location. The following two constraints impose the minimal cycle time of the multi-tour, based on both the travelling, loading and unloading times (Constraint (3.4)) and on the customer imposed frequency constraints (Constraint (3.5)). Constraint (3.6) is the flow conservation constraint for the Z_{jk} variables: when visiting location k, the cumulative demand rate of all remaining customers in the tour is reduced by the demand rate of this location, d_k. Constraint (3.7) links the continuous Z_{jk} variables with the binary X_{jk} variables. The two final constraints determine the maximal cycle time of the multi-tour, based on both the vehicle capacity (Constraint (3.8)) and the customer storage capacities (Constraint (3.9)).

3.9 Conclusion

This chapter presents some solution approaches for the cyclic inventory routing problem. First, two constructive heuristics and an improvement heuristic are proposed, together with heuristic methods for the subproblems of finding tour frequencies and constructing delivery schedules. Then, two metaheuristic solution approaches are developed: a multi-start and a column generation procedure. These methods use and reuse the constructive and improvement heuristics to generate a whole set of solution proposals from which a final solution is then extracted.

The proposed solution approaches are highly generic, since they evaluate many alternatives (from single tours over multi-tours to distribution patterns with a variety of possible tour frequency combinations) and since they do not perform any a priori clustering of customers. In the computational results, presented in Chapter 4, our approach is evaluated.

Chapter 4

Computational results

In this chapter, the value of our solution approach is illustrated in two ways. First, in Section 4.1, the approach is tested on a large set of randomly generated problem instances in a design of experiments. Next, in Section 4.2, the approach is compared to two solution approaches for similar problems found in the literature.

Finally, in Section 4.3, a computational study is reported that illustrates the performance of the best-fit insertion heuristic used for scheduling regular distribution patterns under driving time restrictions.

4.1 Design of experiments

To evaluate the proposed solution approach explained in Chapter 3, a large number of tests were run on problem instances with varying characteristics. During this evaluation, the driving time restriction is always assumed to be active, such that all cycle times are an integer number of days and no vehicle is occupied for more than 8 hours per day. It is also assumed that customers require equidistant deliveries, such that all distribution patterns generated are regular.

The test instances are generated according to a 10×2^5 Factorial Design. This is an experimental design in which 5 factors are considered, each at two levels, and in which, for each possible combination of the factors, 10 instances are generated. The five factors that we selected for the experimental design are given in Table 4.1.

The first factor is the vehicle capacity (VCAP). The two levels of this factor are 100 units and 50 units. For both vehicles, an average speed of 50 km per hour is assumed. The larger vehicle, with a capacity of 100 units, has a fixed cost of 50 euro per hour and a variable transportation cost of 1.2 euro per km, while the smaller vehicle, with a capacity of only 50 units, has a fixed cost of 30

Table 4.1: Factors of the 10×2^5 Factorial Design

Factor	Shorthand	Level '-1'	Level '1'
Vehicle capacity	VCAP	$100u$	$50u$
Customer capacity restriction	CCAP	No	Yes
Holding cost rate	HC	0.10 €/(u·h)	0.01 €/(u·h)
Number of customers	NR	30 – 70	80 – 120
Service area	AREA	75 – 100 km	150 – 175 km

euro per hour and a transportation cost of 1 euro per km. The vehicle capacity factor is included to show that our solution approach not only helps to decide on the fleet size, but can also be used to select the appropriate vehicle type for a particular problem instance.

The second factor is the customer storage capacity restriction (CCAP), with levels 'No' and 'Yes' indicating the inclusion of the restriction or not. When the customer capacity restriction is active, the customer storage capacity is generated randomly such that it can hold between 2 to 10 days of supply. This factor is included to look into the effect of limited customer storage capacity on the total distribution costs.

The third factor is the holding cost rate (HC), which is either 10 or 1 eurocent per unit per hour. This holding cost rate is assumed to be the same for all customers. When considering higher holding costs, e.g. for high value goods or goods that need to be kept in refrigerators, the trade-off between distribution and customer holding costs has to be reconsidered. This factor is included to show that our solution approach is flexible enough to find this new cost trade-off.

The fourth factor is the number of customers (NR). The two levels of this factor are $[30-70]$ and $[80-120]$. This means that the number of customers is either between 30 and 70 or between 80 and 120. This factor is considered to detect potential economies of scale in servicing customers.

The fifth and last factor we consider, is the size of the service area (AREA). All customer locations lie within a circle around the depot. The radius of this circle represents the size of the service area. The service area radius is either between 75 and 100 km, or between 150 and 175 km. This factor is considered to detect potential economies of scale in servicing a certain area.

The last two factors, NR and AREA, are so-called 'randomized factors'. Instead of considering two fixed values for these factors, values are generated randomly in two distinct intervals.

For each combination of the last two factors, the 10 instances are generated as follows. First, the number of customers, NR, is randomly generated between 30 and 70, or between 80 and 120. Then, the radius of the service area, AREA, is randomly generated between 75 and 100 km, or between 150 and 175 km. NR customer locations are then generated randomly within a circle of radius

AREA. For all instances, customer demand rates are randomly generated between 1 to 10 units per day. The depot is always located in the center of the circle. Loading and dispatching a vehicle is assumed to take half an hour ($t_0 = 0.5$h) and cost 20 euro ($\varphi_0 =$ €20), while deliveries at the customers are assumed to take 15 minutes and cost 10 euro ($t_j = 0.25$h, $\varphi_j =$ €10, $\forall j$).

The maximal radius of the service area is 175 km, such that the longest possible tour to a single customer is 350 km. At an average speed of 50 km per hour, this takes 7 hours. With the half hour for dispatching the vehicle and the 15 minutes for unloading, this longest possible tour takes 7.75 hours, which is less than the available 8 hours in a working day. As such, infeasibilities are avoided in our test instances.

The heuristic solution approach described in Chapter 3 was programmed with `MS Visual C++ .NET 2003`, using `ILOG Concert Technology 2.0` and `ILOG CPLEX 9.0` to implement the column generation framework. Computational testing was done on a 2.0 GHz Intel Centrino processor with 1 GB of RAM.

All 320 ($= 10 \cdot 2^5$) instances are solved four times.

1. Using column generation, but restricted to single tours (ST).
2. Using column generation, but restricted to multi-tours (MT).
3. Using column generation and distribution patterns (DP-CG).
4. Using the multi-start procedure and distribution patterns (DP-MS).

The individual results for all these instances are given in Table A.1 of Appendix A. In the discussions below, results are summarized over the various instances. The following solution characteristics are used to explain the various cost trade-offs and how they are obtained.

- total cost rate and the decomposition into its five cost components;
- calculation time;
- number of vehicles and number of tours;
- utilization of the vehicle, i.e. the percentage of time it is being used;
- cumulative average stock level of all customers;
- average number of customers per tour.

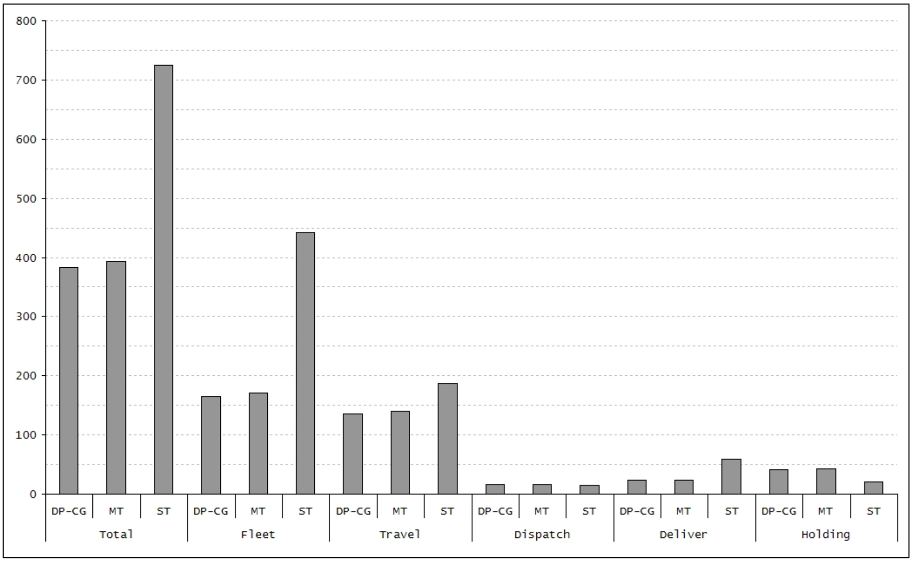


Figure 4.1: Comparing results for the different routing concepts

Table 4.2: Impact of the routing concept

Concept	Total	Fleet	Transport	Dispatch	Deliver	Holding
ST	724.25	442.69	186.67	15.43	59.20	20.27
MT	393.38	171.56	139.50	16.14	23.32	42.86
DP	382.33	165.56	135.98	15.92	23.46	41.41

Concept	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
ST	63.52	11.23	11.23	0.50	375.51	7.13
MT	208.08	4.49	32.22	0.79	987.53	2.93
DP	1278.98	4.33	32.34	0.79	999.27	2.88

4.1.1 Impact of the routing concept

Table 4.2 and Figure 4.1 illustrate the obvious result that generalizing the routing concept from single tours (ST) to multi-tours (MT) and to distribution patterns (DP) results in the ability to achieve increasingly better cost trade-offs. The only drawback is the increased calculation times, but, as indicated before, this is not an issue since we are considering a long-term problem.

For 53 out of our 320 instances, the distribution pattern solution requires one vehicle less than the multi-tour solution. Surprisingly, for 3 instances, it is the other way around. For 243 of the remaining 264 instances, the distribution pattern solution gives a better balance between distribution and holding costs, while for 21 of these instances, the multi-tour solution is better.

The fact that multi-tour solutions and distribution pattern solutions are so close to each other for some instances can be explained as follows. Sometimes, the number of different delivery frequencies in an instance is very limited, possibly due to relatively small differences in demand rates and storage capacity restrictions. If the number of vehicles needed in such instances is large enough such that at least one multi-tour can be constructed for each of the limited set of frequencies, then all frequencies are covered by a multi-tour solution and the extra flexibility of allowing multiple frequencies within a multi-tour is no longer needed. The difference between multi-tour and distribution pattern solutions in our experiments would thus have been more significant if a wider range range of different delivery frequencies was required in the instances (e.g. when the demand rates were between 1 and 50 units per day instead of between 1 and 10).

4.1.2 Impact of the solution approach

In Chapter 3, two metaheuristic solution approaches are presented: a multi-start framework and a column generation framework. To compare these, all 320 instances were solved with both methods. For 169 out of these 320 instances, the column generation procedure gives the best solution, for 147 instances, the multi-start approach is better, and for 4 instances, the same final solution is found. The average total cost rate over all 320 instances is 382.33€/h for the column generation procedure and 382.39€/h for the multi-start procedure. The average CPU-times are respectively 1279 and 1438 seconds. The column generation approach is thus slightly better than the multi-start approach.

As explained in Section 3.8, the column generation is in fact a mathematical programming based version of the multi-start heuristic, with the extra functionality that different solutions from different iterations can be combined in the end in an attempt to find an even better solution. However, for our 320 instances, it happens only 10 times that the final solution of the column generation procedure is actually a combination of distribution patterns from different iterations. This also explains why the column generation is only slightly better than the multi-start procedure.

4.1.3 Significant factor effects and interactions

To evaluate the effects and the interactions of the different factors on the total cost rate of the solution, a stepwise linear regression analysis is performed. In this regression analysis, the five factors and their two-way interactions are evaluated. Higher-level interactions are difficult to interpret and are therefore not considered in the stepwise linear regression. Figure 4.2 shows the final model resulting from this analysis, given by the statistical software `SPlus 7.0`.

In this regression analysis, the software assumes the two levels of all factors to be '-1' and '+1'. For example, for the vehicle capacity factor, a capacity of 100 units corresponds to '-1' and a capacity of 50 units corresponds to '+1'. The

```
 *** Linear Model ***

Call: lm(formula = Total ~ VCAP + CCAP + HC + NR + AREA + VCAP:CCAP
         + CCAP:HC + CCAP:NR + CCAP:AREA + HC:NR + NR:AREA)

Coefficients:
              Value Std.Error t value Pr(>|t|)
(Intercept)  382.34     3.77   101.47   0.0000
       VCAP    7.84     3.77     2.08   0.0382
       CCAP   49.20     3.77    13.06   0.0000
         HC  -47.05     3.77   -12.49   0.0000
         NR  125.76     3.77    33.37   0.0000
       AREA   80.25     3.77    21.30   0.0000
  VCAP:CCAP  -14.27     3.77    -3.79   0.0002
    CCAP:HC   15.30     3.77     4.06   0.0001
    CCAP:NR   12.76     3.77     3.39   0.0008
  CCAP:AREA   17.86     3.77     4.74   0.0000
      HC:NR  -17.30     3.77    -4.59   0.0000
    NR:AREA   33.01     3.77     8.76   0.0000

Residual standard error: 67.41 on 308 degrees of freedom
Multiple R-Squared: 0.87
F-statistic: 187.3 on 11 and 308 degrees of freedom, the p-value is 0
```

Figure 4.2: Output of the regression analysis for the total cost rate

values of the effects and interactions that SPlus outputs are those for going from level '0' to level '+1'. The actual effect of a factor is thus twice this value, since changing a factor means going from level '-1' to level '+1'. For example, the effect of changing the vehicle capacity from 100 units to 50 units is an increase of the total cost rate with $2 \cdot 7.84 = 15.69$€/h.

The output tells us that all five factors have a significant effect and 6 out of 10 factor interactions are also significant. Detailed discussions of these significant effects and interactions are given in the following paragraphs.

Vehicle capacity

```
Coefficients:
             Value Std.Error t value Pr(>|t|)
      VCAP    7.84     3.77     2.08   0.0382
 VCAP:CCAP  -14.27     3.77    -3.79   0.0002
```

The average cost rate is 374.49€/h when using the larger vehicle of 100 units and 390.18€/h when using the smaller vehicle of 50 units. The effect of the vehicle capacity on the total cost rate is thus indeed $2 \cdot 7.84 = 15.69$€/h or 4.19%. However, there is a significant interaction between the vehicle capacity factor and the customer capacity factor, and the magnitude of this interaction is larger than the magnitude of the effect itself. This means that the effect of changing the vehicle capacity is different depending on whether the customer storage capacity restriction is active or not, as can be seen in Figure 4.3. When there is no such restriction (CCAP=No, corresponding to '-1' in `SPlus`), the effect of using the smaller vehicle on the total cost rate is $2 \cdot (7.84 + (-1)(-14.27)) = 2 \cdot (7.84 + 14.27) = 44.23€/\text{h}$. This means that the total cost rate increases by 14.22%. On the other hand, if the customer storage restriction is imposed (CCAP=Yes, corresponding to '+1' in `SPlus`), the effect of using the smaller vehicle on the total cost rate is different: $2 \cdot (7.84 + (+1)(-14.27)) = 2 \cdot (7.84 - 14.27) = -12.86€/\text{h}$, or a decrease of the total cost rate of 2.94%.

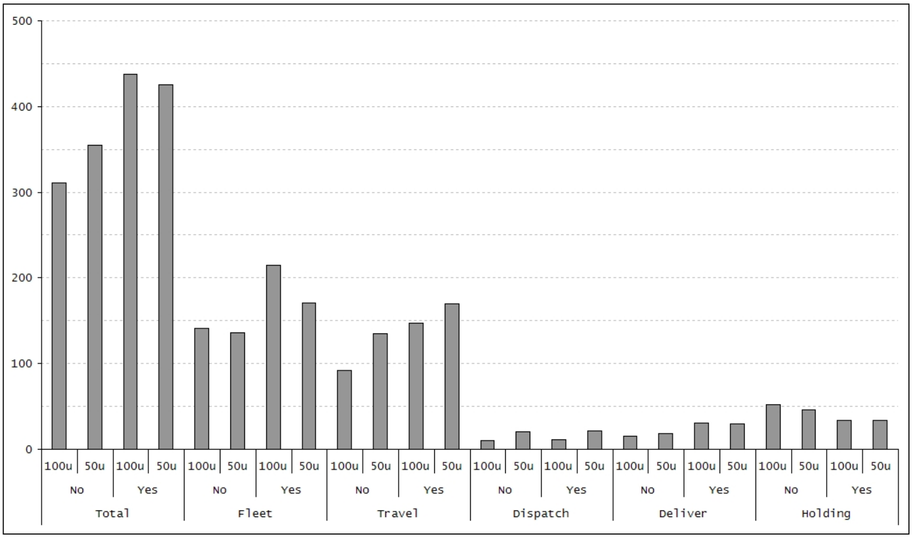


Figure 4.3: Interaction between the factors VCAP and CCAP

Table 4.3 shows the effect of the vehicle capacity on the solution characteristics when there is no customer storage capacity restriction. In this situation, relatively large quantities are delivered, which can be done efficiently by the larger vehicle. When using the smaller vehicle, the number of deliveries increases while the global stock level decreases. Thus, with a smaller vehicle capacity, smaller deliveries are made and less customers are visited per tour. This explains the increase of the dispatching and transportation costs.

The effect of the vehicle capacity when customers impose a storage capacity constraint is shown in Table 4.4. Because delivery quantities are restricted, there is now no longer a difference in the global stock level between the large and small vehicles. When using the smaller vehicles, more routes are travelled to deliver approximately the same quantities. Since the fixed costs and transportation costs are lower, it turns out that the smaller vehicles are the cheaper option.

Table 4.3: Effect of the vehicle capacity when CCAP is inactive

VCAP	Total	Fleet	Transport	Dispatch	Deliver	Holding
100u	311.02	141.25	91.74	10.33	15.52	52.18
50u	355.25	135.75	134.77	20.48	18.12	46.13
Diff	14.22%	-3.89%	46.91%	98.26%	16.79%	-11.61%

VCAP	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
100u	2999.43	2.83	32.81	0.76	1496.41	2.82
50u	1572.82	4.53	48.78	0.80	1118.49	1.68
Diff	-47.56%	60.18%	48.65%	4.56%	-25.26%	-40.26%

Table 4.4: Effect of the vehicle capacity when CCAP is active

VCAP	Total	Fleet	Transport	Dispatch	Deliver	Holding
100u	437.96	215.00	147.21	11.75	30.22	33.78
50u	425.10	170.25	170.21	21.11	29.97	33.56
Diff	-2.94%	-20.81%	15.62%	79.65%	-0.80%	-0.67%

VCAP	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
100u	219.79	4.30	16.86	0.80	691.61	4.55
50u	323.87	5.68	30.93	0.81	690.58	2.48
Diff	47.36%	31.98%	83.40%	1.22%	-0.15%	-45.47%

Customer storage capacity restriction

```
Coefficients:
              Value Std.Error t value Pr(>|t|)
       CCAP   49.20      3.77   13.06   0.0000
  VCAP:CCAP  -14.27      3.77   -3.79   0.0002
    CCAP:HC   15.30      3.77    4.06   0.0001
    CCAP:NR   12.76      3.77    3.39   0.0008
  CCAP:AREA   17.86      3.77    4.74   0.0000
```

In Figure 4.4 and Table 4.5, the global effect of the customer storage capacity restriction on the solution characteristics is shown. When customers impose a storage capacity constraint, the total cost rate increases with $2 \cdot 49.20 = 98.39$€/h, or 29.54%.

Introducing the customer storage capacities results in smaller, more frequent deliveries and thus a lower global stock level. Indeed, the stock level decreases by 47% while the delivery cost rate increases by 79%. Because of the smaller delivery quantities, more customers can be replenished per tour (56%) and less tours are made (41%). However, the increased replenishment frequencies result in an increase of the required number of vehicles (36%) and the transportation costs (40%).

There are some significant interactions between introducing customer storage capacities and other factors in our design of experiments. However, since the

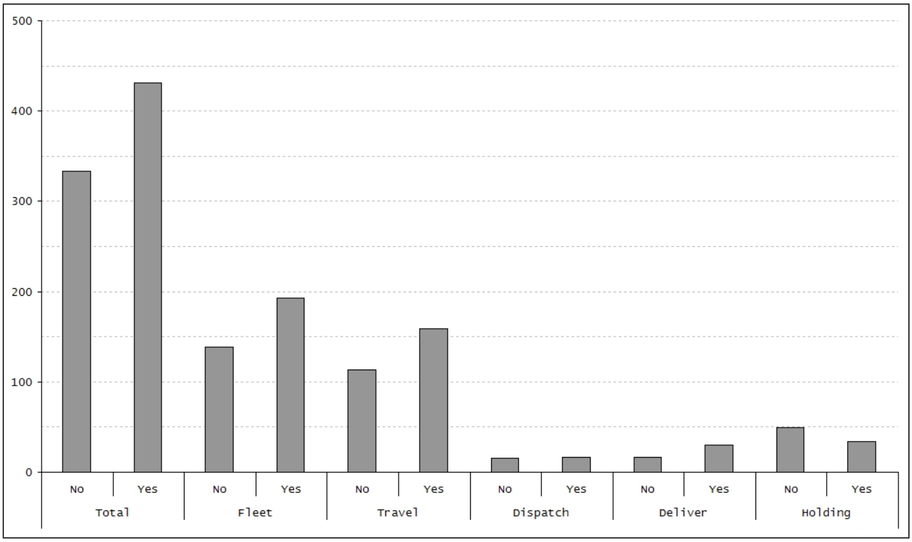


Figure 4.4: Global effect of the customer storage capacity restriction

Table 4.5: Global effect of the customer storage capacity restriction

CCAP	Total	Fleet	Transport	Dispatch	Deliver	Holding
No	333.14	138.50	113.25	15.40	16.82	49.16
Yes	431.53	192.63	158.71	16.43	30.09	33.67
Diff	29.54%	39.08%	40.14%	6.68%	78.90%	-31.51%

CCAP	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
No	2286.12	3.68	40.79	0.78	1307.45	2.25
Yes	271.83	4.99	23.89	0.81	691.09	3.51
Diff	-88.11%	35.71%	-41.43%	3.39%	-47.14%	56.28%

magnitude of these interactions is much smaller than the magnitude of the effect itself, the interactions merely consist in reinforcing or weakening the effect somewhat. The first interaction is with the vehicle capacity. For the large vehicles, the effect of the customer storage capacity is strengthened, while for small vehicles it is weakened. This can also be deducted from Tables 4.3 and 4.4.

The second interaction is with the holding cost factor. This is shown in Figure 4.5 and Tables 4.6 and 4.7. For high holding costs, the effect is a cost increase of $2 \cdot (49.20 + (-1)(15.30)) = 67.79$€/h or 17.14%. The effect is thus somewhat weaker. When holding costs are low, the effect is much larger: $2 \cdot (49.20 + (+1)(15.30)) = 129.01$€/h or 47.64%. However, the story remains the same, but with different magnitudes: smaller delivery quantities, less stock, more tours per customer, less tours, increased replenishment frequencies, more vehicles and more transportation costs. The two other interactions, with the number of customers and the size of the service area, are discussed below.

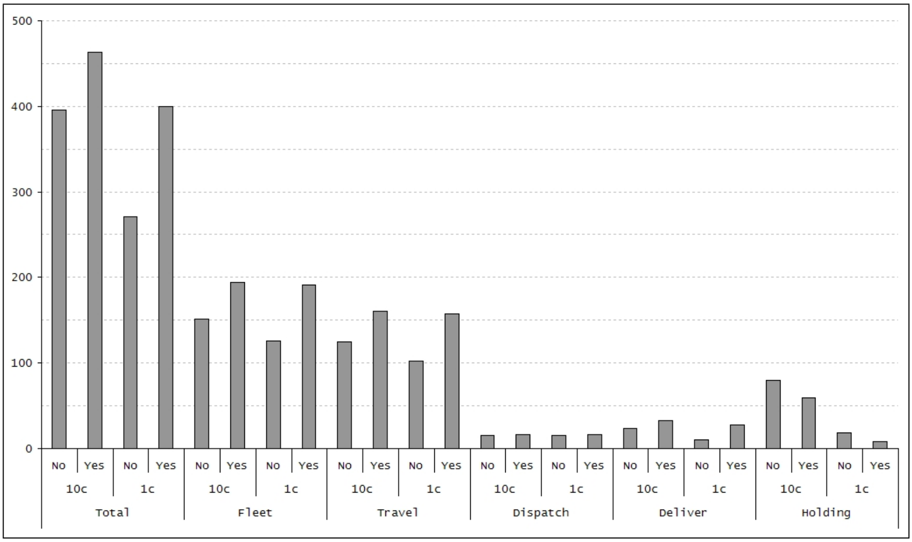


Figure 4.5: Interaction between the factors CCAP and HC

Table 4.6: Effect of CCAP under high holding cost

CCAP	Total	Fleet	Transport	Dispatch	Deliver	Holding
No	395.49	151.63	124.73	15.53	23.43	80.18
Yes	463.28	194.38	160.44	16.33	32.68	59.46
Diff	17.14%	28.19%	28.63%	5.20%	39.47%	-25.84%

CCAP	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
No	424.90	3.99	26.44	0.82	801.81	3.11
Yes	225.50	5.04	20.59	0.82	594.61	3.98
Diff	-46.93%	26.33%	-22.13%	0.21%	-25.84%	27.87%

Table 4.7: Effect of CCAP under low holding cost

CCAP	Total	Fleet	Transport	Dispatch	Deliver	Holding
No	270.78	125.38	101.78	15.28	10.21	18.13
Yes	399.79	190.88	156.99	16.53	27.51	7.88
Diff	47.64%	52.24%	54.25%	8.18%	169.35%	-56.56%

CCAP	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
No	4147.35	3.36	55.15	0.75	1813.09	1.39
Yes	318.16	4.94	27.20	0.80	787.58	3.05
Diff	-92.33%	46.84%	-50.68%	6.87%	-56.56%	120.03%

Another important effect of the customer capacity restriction is seen for the CPU-times. When the restriction is imposed, computation times are much smaller. This can be explained by the fact that the number of tours decreases while the number of vehicles increases. The result is that the number of tours per vehicle is much smaller, such that the number of frequency combination evaluations in the distribution patterns and thus also the number of schedules to be constructed, decreases strongly.

If a distributor is confronted with customers imposing maximal delivery quantities in negotiating contracts, e.g. when setting up a VMI relationship, the distributor can use our solution approach to determine the resulting cost increase. This information can then be used to revise tariffs for these customers.

Holding cost rate

```
Coefficients:
           Value Std.Error t value Pr(>|t|)
       HC -47.05      3.77  -12.49   0.0000
  CCAP:HC  15.30      3.77    4.06   0.0001
    HC:NR -17.30      3.77   -4.59   0.0000
```

In Figure 4.6 and Table 4.8, the global effect of the holding cost rate on the solution characteristics is shown. When changing from a high holding cost to a low holding cost, the total cost rate decreases with $2 \cdot 47.05 = 94.10$€/h, or 21.92%.

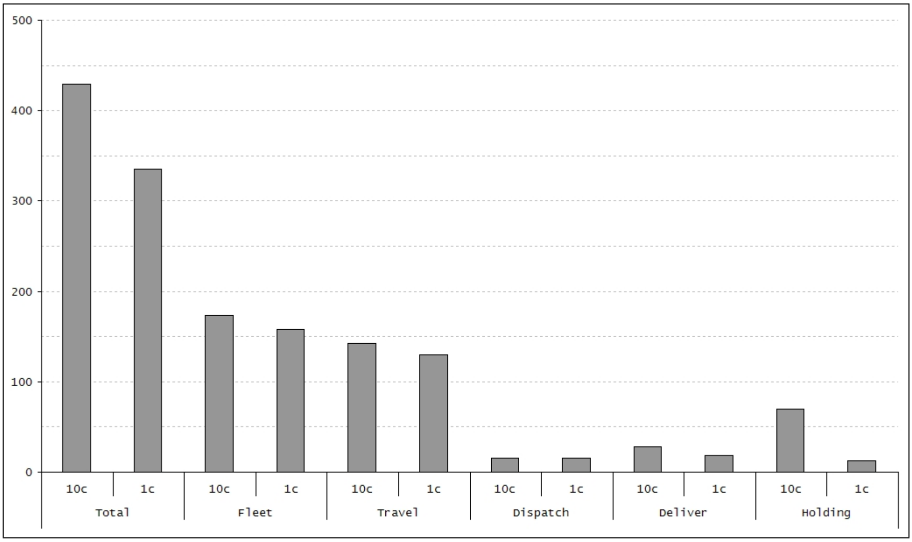


Figure 4.6: Global effect of the holding cost rate parameter

When holding costs are low, larger deliveries are being made. There is indeed an increase of 86% in the global stock level. The larger delivery quantities imply that less customers are visited per tour (37%) and that deliveries are less

Table 4.8: Global effect of the holding cost rate parameter

HC	Total	Fleet	Transport	Dispatch	Deliver	Holding
10c	429.38	173.00	142.58	15.93	28.05	69.82
1c	335.28	158.13	129.38	15.91	18.86	13.00
Diff	-21.92%	-8.60%	-9.26%	-0.12%	-32.76%	-81.38%

HC	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
10c	325.20	4.51	23.51	0.82	698.21	3.54
1c	2232.76	4.15	41.18	0.77	1300.33	2.22
Diff	586.58%	-8.03%	75.12%	-5.32%	86.24%	-37.42%

frequent: the delivery cost rate decreases with 33%. Since deliveries are made less frequent, less vehicles are required (8%) and the travelling costs decrease (9%).

There are two factors that have a significant interaction with the holding cost factor. The first is the customer storage capacity restriction. Imposing this restriction weakens the effect of the changing holding cost. This is obvious, because when holding costs are low, larger delivery quantities are appropriate. However, the customer storage capacity restriction limits the size of the delivery quantities, such that the ideal cost balance cannot be reached.

The other significant interaction is with the number of customers. Increasing the number of customers results in an even larger cost decrease. This can be explained by the fact that a larger set of customers offers are more possibilities to combine customers into efficient tours and distribution patterns.

The holding cost factor also has a strong effect on the CPU-times. For low holding costs, the number of tours per vehicle increases, such that more frequency combinations have to be evaluated and more schedules constructed. This explains the major increase in CPU-times.

Number of customers

```
Coefficients:
             Value  Std.Error  t value  Pr(>|t|)
       NR   125.76       3.77    33.37    0.0000
  CCAP:NR    12.76       3.77     3.39    0.0008
    HC:NR   -17.30       3.77    -4.59    0.0000
  NR:AREA    33.01       3.77     8.76    0.0000
```

In Figure 4.7 and Table 4.9, the effect of the number of customers on the solution characteristics is shown. Of course, an increasing number of customers has an important effect on the computation times. The savings heuristic, used to generate new columns, starts with a separate distribution pattern per customer and then examines all distribution pattern combinations, meaning that computation times increase at least quadratically with the number of customers.

However, the average time needed to solve large problem instances within the solution framework is still only about 40 minutes. Knowing that the time horizon during which the proposed solutions are used is at least a number of weeks and usually even a number of months, these computation times are certainly acceptable.

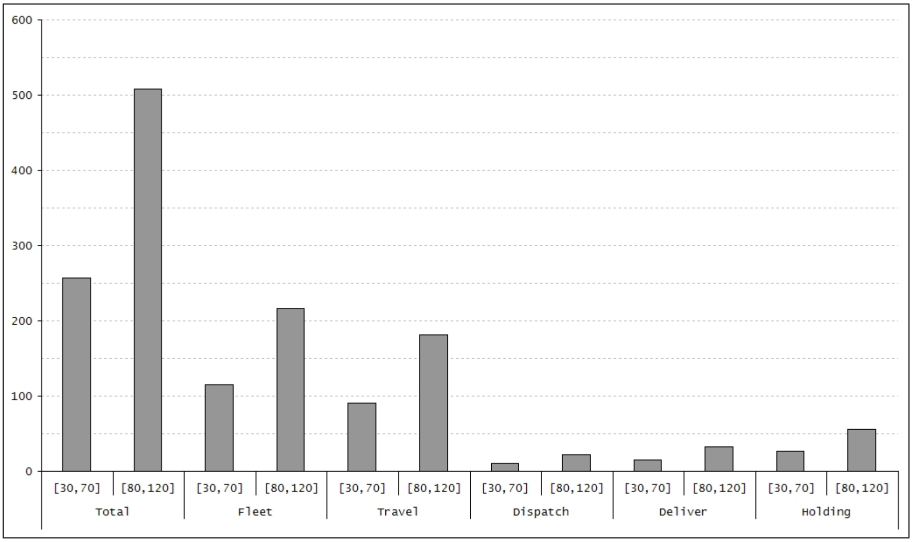


Figure 4.7: Global effect of the number of customers

Table 4.9: Global effect of the number of customers

NR	Total	Fleet	Transport	Dispatch	Deliver	Holding
$[30, 70]$	256.58	114.63	90.15	10.24	14.73	26.84
$[80, 120]$	508.09	216.50	181.82	21.60	32.19	55.99
Diff	98.03%	88.88%	101.69%	110.93%	118.55%	108.61%

NR	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
$[30, 70]$	232.07	2.99	20.88	0.76	632.36	2.81
$[80, 120]$	2325.89	5.68	43.81	0.83	1366.18	2.96
Diff	902.26%	89.96%	109.79%	8.19%	116.04%	5.42%

Although the average number of customers is doubled, the average total cost rate only increases with 98%. This indicates that there is indeed a small economy of scale in serving customers. However, there are three significant interactions with other factors that need to be considered. The first interaction is with the customer storage capacity restriction. As can be seen in Figure 4.8 and Tables 4.10 and 4.11, the total cost rate more than doubles when there are no customer storage capacity restrictions, while this is no longer the case after imposing the restrictions. There is thus only an economy of scale in serving customers if they are imposing storage capacity restrictions. This can be explained by the fact that the presence of the capacity restrictions limits the freedom in designing efficient tours and distribution patterns. Increasing the number of customers then does give more possibilities to combine and results in some relative savings. On the other hand, when the storage capacity restrictions are absent, there is much more freedom and efficient tours and distribution patterns can already be found even for small customer sets.

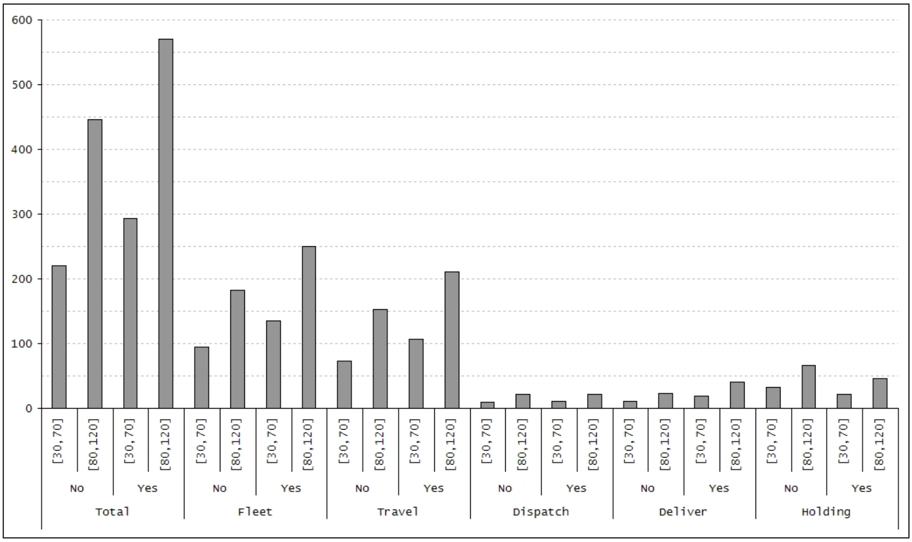


Figure 4.8: Interaction between the factors NR and CCAP

Table 4.10: Effect of the problem size when CCAP is inactive

NR	Total	Fleet	Transport	Dispatch	Deliver	Holding
[30, 70]	220.14	94.50	73.23	9.83	10.33	32.25
[80, 120]	446.13	182.50	153.28	20.98	23.31	66.06
Diff	102.66%	93.12%	109.33%	113.40%	125.63%	104.84%

NR	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
[30, 70]	414.78	2.50	26.46	0.75	834.32	2.16
[80, 120]	4157.47	4.85	55.13	0.81	1780.58	2.34
Diff	902.33%	94.00%	108.31%	8.81%	113.42%	8.09%

The second factor that has a significant interaction with the number of customers, is the holding cost (see Figure 4.9 and Tables 4.12 and 4.13). The story is the same as for the storage capacity restrictions. When holding costs are high, small deliveries are appropriate and it is easier to develop efficient routes, even for small customer sets, such that there is no economy of scale in serving customers. However, when holding costs are high, designing efficient routes is much more difficult because of the larger delivery quantities. In this case, there is an economy of scale for serving more customers, because it offers

Table 4.11: Effect of the problem size when CCAP is active

NR	Total	Fleet	Transport	Dispatch	Deliver	Holding
$[30, 70]$	293.02	134.75	107.07	10.65	19.12	21.43
$[80, 120]$	570.05	250.50	210.36	22.22	41.06	45.91
Diff	94.55%	85.90%	96.47%	108.65%	114.72%	114.28%

NR	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
$[30, 70]$	49.35	3.48	15.30	0.78	430.40	3.45
$[80, 120]$	494.31	6.50	32.49	0.84	951.79	3.58
Diff	901.60%	87.05%	112.34%	7.59%	121.14%	3.74%

more possibilities in combining customers into efficient routes.

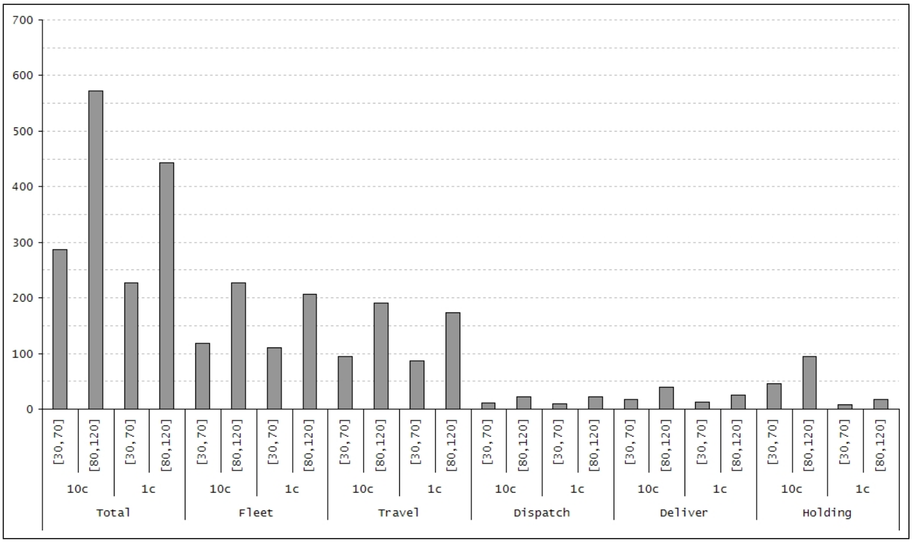


Figure 4.9: Interaction between the factors NR and HC

Table 4.12: Effect of the problem size under high holding costs

NR	Total	Fleet	Transport	Dispatch	Deliver	Holding
$[30, 70]$	286.33	118.88	94.29	10.25	17.33	45.59
$[80, 120]$	572.44	227.13	190.88	21.61	38.78	94.05
Diff	99.92%	91.06%	102.44%	110.73%	123.77%	106.32%

NR	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
$[30, 70]$	66.18	3.09	15.75	0.79	455.87	3.39
$[80, 120]$	584.22	5.94	31.28	0.85	940.54	3.70
Diff	782.80%	92.31%	98.57%	7.75%	106.32%	9.19%

The third and last factor having a significant interaction with the number of

Table 4.13: Effect of the problem size under low holding costs

NR	Total	Fleet	Transport	Dispatch	Deliver	Holding
[30, 70]	226.82	110.38	86.01	10.23	12.13	8.09
[80, 120]	443.75	205.88	172.76	21.59	25.60	17.92
Diff	95.63%	86.52%	100.87%	111.14%	111.08%	121.53%

NR	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
[30, 70]	397.95	2.89	26.01	0.74	808.85	2.22
[80, 120]	4067.56	5.41	56.34	0.80	1791.82	2.21
Diff	922.12%	87.45%	116.58%	8.65%	121.53%	-0.34%

customers, shown in Figure 4.10, is the size of the service area. Comparing Tables 4.14 and 4.15 reveals that, when the service area is large, the tours are longer and therefore, replenishment frequencies are decreased to prevent the transportation costs from increasing too much. This leads to larger deliver quantities, which are more difficult to combine into efficient routes, especially because the average distance between customers is larger. The result is that there is no economy of scale in serving customers in a large service area. However, in a small service area, replenishment quantities are smaller and customers are located closer to each other, such that there is indeed an economy of scale in serving customers in a smaller service area.

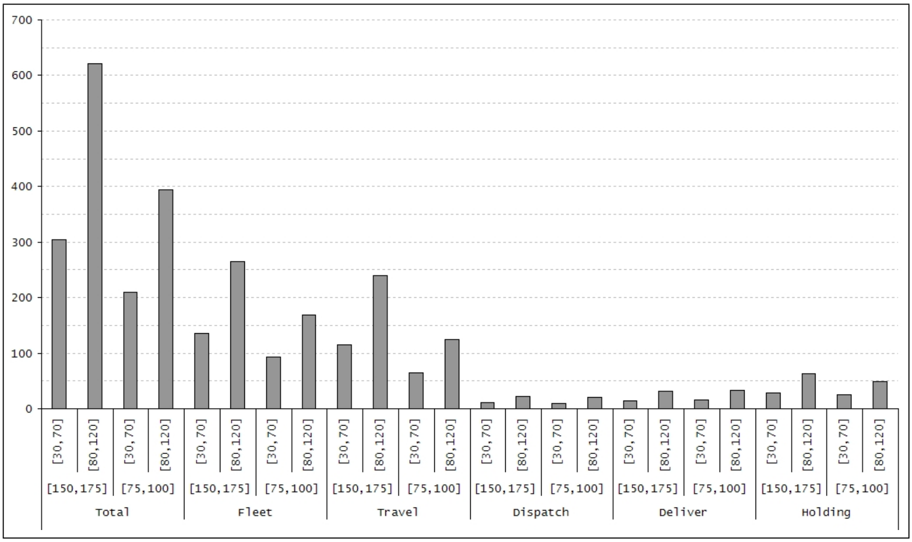


Figure 4.10: Interaction between the factors NR and AREA

Table 4.14: Effect of the problem size in a small service area

NR	Total	Fleet	Transport	Dispatch	Deliver	Holding
[30, 70]	209.34	93.75	64.64	10.10	16.01	24.83
[80, 120]	394.83	168.50	124.38	20.45	32.64	48.86
Diff	88.61%	79.73%	92.42%	102.45%	103.79%	96.79%

NR	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
[30, 70]	325.95	2.45	19.64	0.76	603.20	3.18
[80, 120]	2753.49	4.43	39.28	0.82	1227.78	3.21
Diff	744.75%	80.61%	100.00%	8.45%	103.54%	0.97%

Table 4.15: Effect of the problem size in a large service area

NR	Total	Fleet	Transport	Dispatch	Deliver	Holding
[30, 70]	303.82	135.50	115.65	10.38	13.44	28.85
[80, 120]	621.35	264.50	239.26	22.74	31.74	63.11
Diff	104.52%	95.20%	106.88%	119.19%	136.12%	118.79%

NR	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
[30, 70]	138.18	3.53	22.13	0.77	661.52	2.43
[80, 120]	1898.28	6.93	48.34	0.83	1504.58	2.70
Diff	1273.80%	96.45%	118.47%	7.93%	127.44%	11.24%

Size of the service area

```
Coefficients:
                Value  Std.Error  t value  Pr(>|t|)
       AREA     80.25       3.77    21.30    0.0000
  CCAP:AREA     17.86       3.77     4.74    0.0000
    NR:AREA     33.01       3.77     8.76    0.0000
```

In Figure 4.11 and Table 4.16, the effect of the size of the service area on the solution characteristics is shown. In the large service area, more vehicles are needed and more distance needs to be travelled, because of the increased average distance between customers. To balance this with the other cost rate components, replenishment frequencies go down, such that larger quantities are being delivered and less customers are visited per tour. The average total cost increase is 53.13%. There is thus a strong economy of scale in serving a customer area. Making the radius of the area twice as big, and hence the service area surface four times as big, leads to a cost increase of only little more than 50%.

The decrease in CPU-times is again explained by the fact that the average number of tours per vehicle is smaller in large service areas, limiting the necessary frequency combination evaluations and delivery schedule constructions.

Two factors have a significant interaction with the size of the service area. The first of these is the customer storage capacity restriction (see Figure 4.12

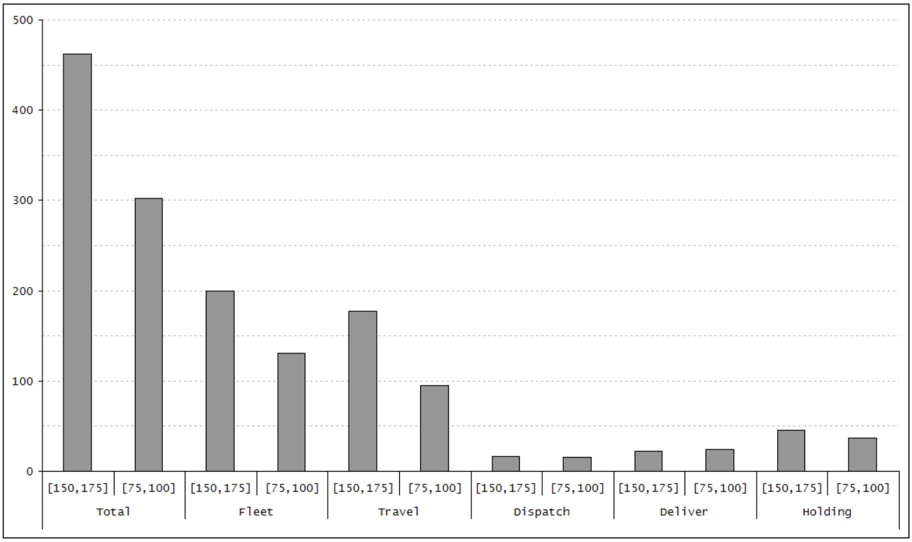


Figure 4.11: Global effect of the service area size

Table 4.16: Global effect of the service area size

AREA	Total	Fleet	Transport	Dispatch	Deliver	Holding
$[75, 100]$	302.08	131.13	94.51	15.28	24.32	36.85
$[150, 175]$	462.59	200.00	177.46	16.56	22.59	45.98
Diff	53.13%	52.53%	87.77%	8.38%	-7.13%	24.78%

AREA	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
$[75, 100]$	1539.72	3.44	29.46	0.79	915.49	3.20
$[150, 175]$	1018.23	5.23	35.23	0.80	1083.05	2.56
Diff	-33.87%	52.00%	19.61%	1.16%	18.30%	-19.84%

and Tables 4.17 and 4.18). If this restriction is active, the increase of delivery quantities is limited, such that it is difficult to balance distribution costs with holding costs. Without customer storage capacity restrictions, a better cost trade-off can be achieved, making the economy of scale in serving a customer area even larger. The second interaction, with the number of customers, is discussed in the previous paragraph (see Figure 4.10 and Tables 4.14 and 4.15).

4.2 Comparison to other heuristics

In this section, our heuristic solution approach is compared to the results of Viswanathan and Mathur (1997) [30], and Sindhuchao et al.(2005) [28].

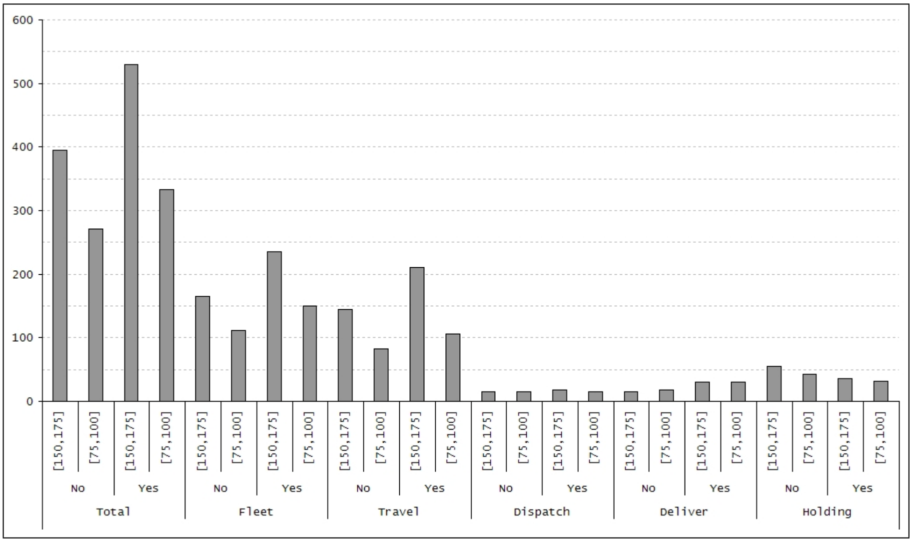


Figure 4.12: Interaction between the factors AREA and CCAP

Table 4.17: Effect of the service area size when CCAP is inactive

AREA	Total	Fleet	Transport	Dispatch	Deliver	Holding
[75, 100]	270.74	112.13	82.65	15.11	18.19	42.66
[150, 175]	395.53	164.88	143.86	15.70	15.45	55.65
Diff	46.09%	47.05%	74.07%	3.86%	-15.05%	30.43%

AREA	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
[75, 100]	2778.77	2.96	36.89	0.79	1173.49	2.52
[150, 175]	1793.48	4.39	44.70	0.77	1441.40	1.98
Diff	-35.46%	48.10%	21.18%	-2.14%	22.83%	-21.70%

Table 4.18: Effect of the service area size when CCAP is active

AREA	Total	Fleet	Transport	Dispatch	Deliver	Holding
[75, 100]	333.43	150.13	106.37	15.45	30.46	31.03
[150, 175]	529.64	235.13	211.06	17.42	29.73	36.31
Diff	58.85%	56.62%	98.42%	12.79%	-2.39%	17.02%

AREA	CPU-time	nrVeh	nrTours	Utilization	Stock	Cust/tour
[75, 100]	300.67	3.91	22.03	0.79	657.49	3.88
[150, 175]	242.98	6.06	25.76	0.83	724.70	3.15
Diff	-19.19%	54.95%	16.97%	4.45%	10.22%	-18.62%

Viswanathan and Mathur

Viswanathan and Mathur [30] developed a heuristic that generates a so-called stationary nested joint replenishment policy (SNJRP). They term a policy to be *stationary* if replenishments are made at equally spaced points in time. This is the equivalent of what we have called *regular*. A nested policy means that if the replenishment interval of a given customer is larger than that of another customer served by the same vehicle, the former is a multiple of the latter. Thus, the nestedness corresponds to our delivery frequencies, because not all customers visited by the same vehicle have the same replenishment interval.

As in most papers found in the literature on cyclic inventory routing, a fixed vehicle cost is not considered in the cost structure of Viswanathan and Mathur. As a result, the assignment of tours to vehicles is not taken into account and the assumption is made that there is a separate vehicle per tour.

For the cycle times of the tours, only vehicle capacity is used as a constraining element. This means that, compared to our model, the restrictions following from (i) travelling, loading and unloading times, (ii) customer imposed maximal delivery frequencies, and (iii) limited customer storage capacities, are discarded. In other words, a minimal cycle time for the (nested) tours is not being considered.

To compare our heuristic method to that of Viswanathan and Mathur, the following adaptations were made to our model as presented in Chapter 2:

- The fixed vehicle cost is discarded: $\psi = 0$.
- The vehicle speed is very high ($\nu = \infty$), such that the minimal cycle time is always negligible.

The problem instances that Viswanathan and Mathur used in their computational testing were not available from the authors, so we reused the set of problem instances that were generated for our design of experiments (see Section 4.1), excluding the instances with customer storage capacity restrictions. Because a common base cycle time is used in the solution approach of Viswanathan and Mathur, we impose a 1 day common base cycle time for replenishment in these tests by activating the driving time restriction in our solution approach.

Figure 4.13 and Table 4.19 show the average improvement of the solutions obtained with our solution method using distribution patterns (DP) over those obtained with the stationary nested joint replenishment heuristic (SNJRH).

For all problem instances, solutions were found that are cheaper than those proposed by Viswanathan and Mathur. On average, there is a decrease in cost of no less than 7.4%. This is mostly due to the more general routing concept that we are using. These results show that allowing vehicles to make multiple tours gives the opportunity to use vehicle capacity much more efficiently and find much better cost trade-offs, even without considering fixed vehicle costs.

Our solution approach is thus capable of finding better trade-offs between distribution and customer holding costs due to the generalized model. When considering fixed vehicle costs, the model of Viswanathan and Mathur becomes

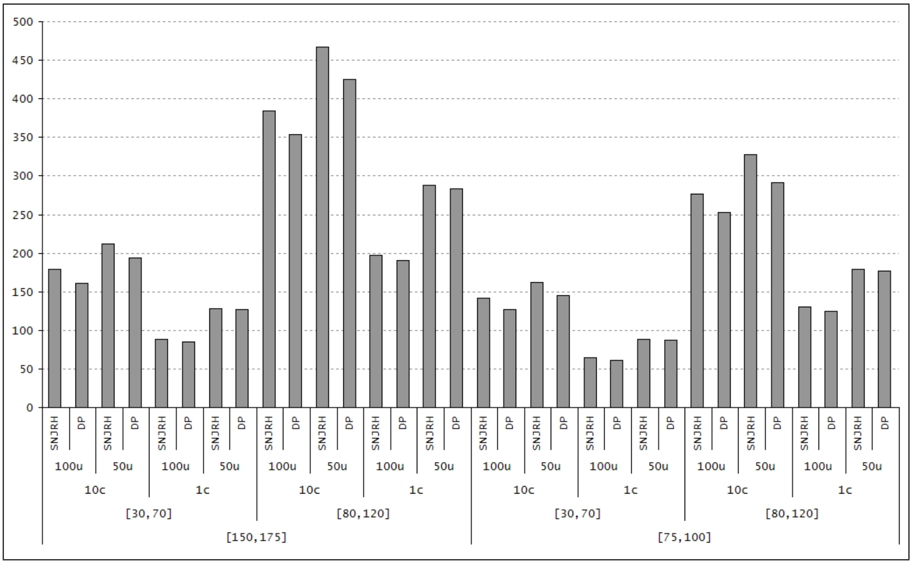


Figure 4.13: Comparing our solutions to that of the stationary nested joint replenishment heuristic

Table 4.19: Improvement of our heuristic over SNJRH

VCAP	HC	NR	AREA	DP	SNJR	Δ
100u	10c	[30,70]	[150,175]	178.82	161.49	10.7%
50u	10c	[30,70]	[150,175]	211.74	193.52	9.4%
100u	1c	[30,70]	[150,175]	88.90	85.57	3.9%
50u	1c	[30,70]	[150,175]	128.62	127.35	1.0%
100u	10c	[80,120]	[150,175]	384.88	354.06	8.7%
50u	10c	[80,120]	[150,175]	466.82	425.05	9.8%
100u	1c	[80,120]	[150,175]	197.75	191.02	3.5%
50u	1c	[80,120]	[150,175]	287.83	283.82	1.4%
100u	10c	[30,70]	[75,100]	141.30	127.15	11.1%
50u	10c	[30,70]	[75,100]	162.35	145.46	11.6%
100u	1c	[30,70]	[75,100]	65.08	61.43	5.9%
50u	1c	[30,70]	[75,100]	88.39	86.87	1.7%
100u	10c	[80,120]	[75,100]	276.59	253.39	9.2%
50u	10c	[80,120]	[75,100]	327.33	291.23	12.4%
100u	1c	[80,120]	[75,100]	130.67	124.47	5.0%
50u	1c	[80,120]	[75,100]	179.58	176.88	1.5%
Average				207.29	193.05	7.4%

obsolete, while our approach is still valid and is capable of finding three-way cost trade-offs between distribution, customer holding and fixed vehicle costs.

Sindhuchao et al.

Sindhuchao et al. [28] develop a branch-and-price algorithm that finds the optimal solution for a set of very small problem instances. Again, fixed vehicle costs are not considered in the cost structure, such that only solutions with a single tour per vehicle are obtained.

In the literature on cyclic inventory routing, Sindhuchao et al. are the only ones that consider a lower bound on tour cycle times. However, this minimal cycle time is not the natural one arising due to the travelling, loading and unloading times. Instead, they impose a vehicle frequency constraint F, stating that a vehicle cannot make more than F tours per period.

To compare our heuristic method to the results of Sindhuchao et al., the following adaptations had to be made:

- The fixed vehicle cost is discarded: $\psi = 0$.
- The vehicle speed is very high ($\nu = \infty$), such that the original minimal cycle time is negligible.
- The new minimal cycle time due to the vehicle frequency constraint is imposed as follows: $\sum_{i=1..n} k_i < F \cdot T$, with F the maximum number of tours per period. This gives $T_{min} = \sum_{i=1..n} k_i / F$.

Our adjusted heuristic solution approach was then applied to the set of very small problem instances for which Sindhuchao et al. found the optimum. The results are shown in Table 4.20.

Table 4.20: Optimal cost vs. obtained solutions

	Optimum	Solution	Gap
1	2778.1	2856.0	2.80%
2	2645.8	2682.7	1.40%
3	2598.6	2629.5	1.19%
4	2761.2	2804.9	1.58%
5	2726.0	2726.0	0.00%
6	2699.3	2707.2	0.29%
7	2526.7	2564.8	1.51%
8	2426.2	2449.1	0.94%
9	2577.2	2606.9	1.15%
10	2825.5	2865.4	1.41%
Avg	2656.5	2689.2	1.23%

It can be seen that our heuristic performs very well on these 10 instances. The average gap between our results and the optimal solutions is only 1.23% and the largest gap is 2.80%. This again confirms the power of our approach. While it can handle much more general versions of the cyclic inventory routing problem, it still manages to get close to the optimum for this specific version.

Next, we took the comparison one step further. We replaced the unrealistic definition of the minimal cycle time based on the vehicle frequency constraint by our more realistic definition based on the required travel times. In the solutions for the 10 test problems, the maximal distance a vehicle travels is 616 km per period. Therefore, we restored our original definition of the minimal cycle time, using a vehicle speed of 616 km per period and solved the same instances again. Table 4.21 shows the results of these test. It turns out that for all out of these instances, a solution can be found that is cheaper than the 'optimum' reported by Sindhuchao et al., with an average improvement of 4%.

Table 4.21: 'Optimal' cost vs. improved solutions

	'Optimum'	Solution	Gap
1	2778.1	2722.4	-2.01%
2	2645.8	2590.2	-2.10%
3	2598.6	2597.1	-0.06%
4	2761.2	2533.7	-8.24%
5	2726.0	2678.0	-1.76%
6	2699.3	2492.0	-7.68%
7	2526.7	2435.7	-3.60%
8	2426.2	2366.8	-2.45%
9	2577.2	2507.4	-2.71%
10	2825.5	2580.7	-8.66%
Avg	2656.5	2550.4	-3.99%

4.3 Performance of the scheduling heuristic

In this section, the best-fit insertion heuristic for scheduling regular distribution patterns under driving time restrictions is evaluated. Similar experiments as those reported in this section can be conducted for the other variants of the scheduling problem, but are not reported here.

Although the problem of cyclically scheduling regular distribution patterns under driving time restrictions is NP-hard, a branch-and-bound algorithm is used for evaluating the heuristic. For the branch-and-bound algorithm, Constraint (4.1) is added to the model given in Table 2.13. This constraint explicitly specifies the equidistance constraint on the X-variables that is implicit in the original model through Constraints (2.57) and (2.58). The constraints states that (i) if the l'th iteration of tour i is made on day t, then the $(l+1)$'th iteration is made $\frac{T_C}{k_i}$ days later, and (ii) if the l'th iteration of tour i is not made on day t, then the $(l+1)$'th iteration is not made $\frac{T_C}{k_i}$ days later.

$$X_{il}^{t} = X_{i,l+1}^{t+\frac{T_S}{k_i}} \quad \begin{cases} i = 1..n \\ l = 1..k_i - 1 \\ t = 1..l\frac{T_S}{k_i} \end{cases} \tag{4.1}$$

However, the above constraint is non-linear because the variable T_S is used as an index. But since we know that T_S is an integer multiple of the base cycle time B between T_{MIN} and T_{MAX}, a linear version of the constraint can be written as follows.

$$X_{il}^{t} \leq \sum_{m=T_{MIN}/B}^{T_{MAX}/B} X_{i,l+1}^{t+m\frac{B}{k_i}} \quad \begin{cases} i = 1..n \\ l = 1..k_i - 1 \\ t = 1..l\frac{T_{MAX}}{k_i} \end{cases} \tag{4.2}$$

Adding Constraint (4.2) to the model helps in reducing the calculation times for the branch-and-bound procedure. To evaluate the proposed heuristic, a set of 100 randomly generated instances is created, for which the optimal solution is determined and then compared to the heuristic solution. To further speed up the branch-and-bound algorithm that gives the optimal solution, the heuristic is used as an initial solution.

The test instances are generated as follows. First, the number of tours n is generated randomly between 3 and 20. Then, the tour frequencies k_i are generated randomly from the set of frequencies $\{1, 2, 3, 4, 6, 8\}$ and the tour durations TD_i are generated randomly between 1 and 8 hours. Frequencies of 5 and 7 are not considered because this would increase the base cycle time dramatically, making the schedules very sparse, which is very unrealistic. Customer demand rates are not considered in the experiments, such that the maximal cycle time T_{MAX} is not defined and a feasible schedule can be found for all instances. In the branch-and-bound algorithm, the cycle time that is found by the insertion heuristic is used as T_{MAX}.

The heuristic procedure and the branch-and-bound algorithm are both implemented in `Microsoft Visual C++`, the latter by building the model with `ILOG Concert Technology` and solving the model using the `ILOG Cplex` solver.

Table 4.22 shows the results of our computational experiments. For all instances i, the number of tours n and the base cycle time B in that instance are displayed. Then, the cycle time that the heuristic solution approach returns (Sol) and the optimal cycle time found by the branch-and-bound algorithm (Opt) are reported. The final column displays the gap between the heuristic and the optimal solution. This gap is defined as the difference between both, divided by the base cycle time B. The gap thus reports how many times the cycle time was increased above the optimum in the heuristic.

Our insertion heuristic finds the optimal solution for 79 out of 100 instances. For 20 instances, the gap is 1, meaning that the heuristic had to increase

Table 4.22: Computational results for the 100 test instances

i	n	B	Sol	Opt	Gap	i	n	B	Sol	Opt	Gap
1	9	12	36	36	0	51	18	24	96	72	1
2	3	12	24	24	0	52	10	12	24	24	0
3	15	24	72	72	0	53	17	24	72	48	1
4	9	24	48	48	0	54	6	24	48	48	0
5	9	24	24	24	0	55	17	24	72	72	0
6	5	24	48	48	0	56	9	24	48	48	0
7	19	24	72	48	1	57	10	24	48	48	0
8	4	24	48	48	0	58	6	8	24	24	0
9	14	24	96	72	1	59	12	24	72	72	0
10	17	24	72	72	0	60	16	24	48	48	0
11	20	24	96	96	0	61	5	6	18	18	0
12	15	12	60	48	1	62	5	24	48	48	0
13	8	24	48	48	0	63	12	24	48	48	0
14	9	24	48	48	0	64	4	4	12	12	0
15	15	24	96	72	1	65	10	24	48	48	0
16	4	6	12	12	0	66	17	24	48	48	0
17	13	24	48	48	0	67	13	24	48	48	0
18	18	24	96	72	1	68	9	24	72	48	1
19	7	24	48	48	0	69	15	12	36	36	0
20	10	24	48	48	0	70	6	24	24	24	0
21	10	24	48	48	0	71	9	24	48	48	0
22	14	24	48	48	0	72	4	8	16	16	0
23	7	24	48	48	0	73	8	24	48	24	1
24	8	24	48	48	0	74	4	6	12	12	0
25	17	24	72	72	0	75	6	24	48	48	0
26	17	24	72	72	0	76	16	24	48	48	0
27	8	24	48	48	0	77	13	24	48	48	0
28	14	24	72	72	0	78	6	24	48	24	1
29	19	24	72	72	0	79	13	12	24	24	0
30	13	24	24	24	0	80	18	24	48	48	0
31	3	4	4	4	0	81	6	8	24	24	0
32	5	24	48	48	0	82	20	24	96	72	1
33	19	24	72	72	0	83	5	24	48	48	0
34	14	12	60	48	1	84	11	24	48	48	0
35	4	12	24	24	0	85	6	24	48	48	0
36	6	24	48	48	0	86	5	12	24	24	0
37	8	24	24	24	0	87	13	24	24	24	0
38	19	24	96	72	1	88	12	24	72	72	0
39	10	24	72	48	1	89	14	24	72	48	1
40	18	24	72	48	1	90	18	24	48	48	0
41	15	24	48	48	0	91	3	6	6	6	0
42	10	24	48	48	0	92	18	24	96	48	2
43	20	24	96	96*	0	93	3	4	8	8	0
44	14	24	72	48	1	94	16	24	72	72	0
45	6	24	48	48	0	95	12	24	48	48	0
46	6	8	24	24	0	96	17	24	72	72	0
47	8	24	48	48	0	97	14	24	72	48	1
48	14	24	48	48	0	98	13	24	48	48	0
49	15	24	72	48	1	99	3	8	16	16	0
50	16	24	96	72	1	100	11	24	48	48	0

the cycle time once too much before finding a feasible solution. For only one instance, number 92, the heuristic solution is two times the base cycle time above the optimum. Note that the instances for which the heuristic does not find the optimum are instances with a relatively large number of tours. Instance 43 is marked with an asterisk because the branch-and-bound could not prove the optimality even after 12 hours.

4.4 Conclusion

This chapter proves the value of our modelling and solution approach. First, the design of experiments shows that our solution approach is highly generic and flexible in finding cost trade-offs for problem instances with widely varying characteristics. Next, the comparison to two approaches found in the literature shows that (i) the solutions we obtain are very close to optimality, and (ii) the existing heuristic solution approaches from the literature are outperformed by our approach that uses the more general routing concept of distribution patterns. Finally, another set of experiments shows the effectiveness of the best-fit insertion heuristic for regular scheduling under driving time restrictions.

Chapter 5

Real-life application

This chapter reports on a real-life application of distribution optimization, in which we used our cyclic solution approach. The application deals with setting up a new distribution strategy for distributing low-value goods in the Benelux, from a single warehouse to a set of customers with stable consumption rates.

5.1 Problem setting

This study was done for the Sales & Marketing office of a paper goods producer, selling to customers in Flanders and the Netherlands. At the time of the study, Flemish and Dutch customers were delivered by two different logistics service providers (LSP's), from two different warehouses. Both warehouses are replenished separately by direct deliveries from the manufacturing site in Scandinavia. The S&M office ordered this study because it wanted to minimize its total logistics costs. In setting up the new distribution strategy, it was obvious that only a single warehouse is needed, instead of separate warehouses. This will of course reduce the costs for replenishment from the manufacturing site. However, the focus of this study is on the next step, i.e. how to organize the distribution from this single warehouse to all customers in both Flanders and the Netherlands. Since most customers have a long-term commitment and stable consumption rates, it was decided to set up a cyclic distribution strategy. Thus, the objective of this study is two-fold: (i) finding the optimal location for a single Benelux warehouse, and (ii) designing cost efficient vehicles routes from this warehouse to the customers.

In the case under consideration, multiple products are being distributed. To deal with multiple products, two modelling strategies are available: (i) represent a customer consuming p different products by a set of p (dummy) customers, all located in the same place, and each consuming one of the products; (ii) aggregate the demand rates of the different products per customer. In the first strategy, different products can be delivered in separate tours (i.e. split delivery) and a separate delivery handling cost is incurred for each product that is delivered to a customer. Moreover, this approach significantly increases the problem size. In the case under consideration, over 250 different products are being distributed to over 100 customers.

Since most of the customers are small and do not have dedicated staff for delivery handling, the S&M office wishes to keep the replenishment strategy as simple as possible for the sake of the customers. Therefore, split deliveries are not an option. The second strategy for dealing with multiple products is thus adopted, and demand rates of the different products are aggregated per customer.

5.2 Solution approach

The S&M office is only interested in minimizing the distribution costs and does not bother with inventory costs at the customers. As such, holding costs are not included in the study, but the customers do have their impact by imposing a storage capacity constraint. In fact, discarding the customer holding costs has no impact on the solution, since the considered goods are all low-value and have very small holding costs. The resulting EOQ cycle times and delivery quantities are thus very large, unfeasibly large because of limited vehicle and customer storage capacities. The best feasible cycle time is therefore always the maximal cycle time, whether holding costs are being taken into account or not.

The driving time is restricted to 8 hours per day, and 5 days a week. As a result, cycle times are always an integer number of weeks, and customers are always replenished on the same weekday, but not necessarily every week. Some customers impose an extra restriction by specifying their delivery day: replenishment has to be on a specific day of the week, but again, not necessarily every week. When inserting a customer with a pre-determined delivery day into a tour, the weekday to which this tour has to be assigned is also fixed. This also means that customers with different pre-determined delivery days cannot be in the same tour. Due to the inclusion of pre-determined delivery days for some customers, the heuristic for scheduling under driving time restrictions as developed in Section 3.6, has to be adjusted.

The fact that the maximal cycle time of a distribution pattern is always the best feasible cycle time also affects the scheduling subproblem. It is no longer necessary to find a schedule with minimal makespan, because the schedule with makespan equal to the maximal cycle time will always be the one that is actually implemented. The scheduling heuristic is then as follows.

1. The schedule time T_S is equal to the maximal cycle time T_{MAX}.

2. If all tours have been added to the schedule, the procedure stops here and a feasible schedule is found. Else, determine the next tour to be added to the schedule. This is the tour with a pre-determined delivery day and with the highest frequency that is different from the frequency of the previously added tour (and with the longest travel time in case of a tie). If all tours with a pre-determined delivery day have been added to the schedule, the next tour to be inserted is the tour with the highest frequency that is different from the frequency of the previously added tour (and with the longest travel time in case of a tie). The selected tour i has to be made k_i times within a cycle of T_S days, such that the first iteration has to be on a day in the interval $[1, \frac{T_S}{k_i}]$.

3. For t^* from 1 to $\frac{T_S}{k_i}$.

   a. If the selected tour has a pre-determined delivery day and t^* does not correspond to this day, go to the next t^*.

   b. The days on which tour i is made are: $t^* + (k-1)\frac{T_S}{k_i}$ (with $k = 1..k_i$). If the 8-hour restriction is violated on any of these days, go to the next t^*.

   c. For each of the days $t^* + (k-1)\frac{T_S}{k_i}$ (with $k = 1..k_i$), sum the time remaining on these days after inserting tour i. The t^* for which this cumulative remaining time is minimal, will eventually be selected.

4. If no t^* can be found that results in a feasible schedule, the process stops here and the distribution pattern is infeasible. Else, return to Step 2.

Since the S&M office is not intending to invest in (expensive) optimization software for column generation and solving the final master partitioning problem, the multi-start solution approach is used.

For setting up the cyclic distribution strategy, 112 customers are considered, consuming 79 pallets of product per week. From these customers, 32 are located in the Netherlands and 80 in Flanders. The Dutch customers are relatively big, since they demand 38 pallets per week, while the Belgian customers consume 41 pallets per week. The customers considered are those with a stable aggregated demand rate. For most of the customers, this assumption is valid. However, there are a number of 'opportunistic' customers, who scan the market for the cheapest product prices and then order big volumes. These customers are very unstable and unpredictable, and can therefore not be included in the cyclic scheme.

In Table 5.1, a sample of the input data is shown. For deliveries, it is assumed that it takes 15 minutes (0.25h) and costs 5 euro. Some customers impose their delivery day: e.g. customer F203 can only receive deliveries on Thursday. The aggregated demand rate is given in pallets per week, and the storage capacity is in pallets. Maximal cycle times are then easily derived: e.g. customer N261 consumes 0.32 pallets per week and has room for only one pallet, meaning that it needs to be replenished at least every three weeks.

Table 5.1: Example of input data

Customer	Dem	Cap	DelT	DelC	DelDay	City
...						
N019	0.4796	1	0.25	5	Tue	Eindhoven
N093	1.1871	2	0.25	5	Tue	Den Bosch
N180	0.0279	1	0.25	5	Any	Amsterdam
N261	0.3224	1	0.25	5	Any	Rotterdam
F063	0.6735	3	0.25	5	Any	Brugge
F085	0.3109	1	0.25	5	Any	Mechelen
F130	0.0789	1	0.25	5	Any	Gent
F203	0.7473	3	0.25	5	Thu	Antwerpen
F230	0.2933	1	0.25	5	Any	Hasselt
F234	0.1892	1	0.25	5	Any	Oostende
...						

5.3 Results

To find the optimal location for the warehouse, a center-of-gravity approach was used within the `Arcview` GIS software. The proposed location is near the Dutch-Flemish border, along the highway. The same software was then used to calculate travel times between the customers and the warehouse.

To organize the distribution from the warehouse, two strategies are studied: in-house distribution and outsourcing to an LSP. If the S&M office would decide to organize the distribution activities in-house, then the cost structure has to include a fixed vehicle cost, because the S&M office has to acquire its own vehicle fleet. For outsourcing to an LSP, however, the cost structure consists of variable costs only. The variable cost parameter values for dispatching, delivering and transportation are then usually higher than for the in-house case. However, since tariffs of LSP's from the new warehouse location were not available at the time of the study, we use the same values for the variable cost parameters in all experiments. Once these tariffs are available, the experiments can be repeated to determine the actual costs. The objective of this study is to look into the different solution structures.

The multi-start savings heuristic was run twice for both strategies, with two different types of vehicles. The first truck type has a capacity of 10 pallets, a fixed cost of 1000 euro per week, and a variable transportation cost of 13 euro per hour. The second truck type has a capacity of 15 pallets, a fixed cost of 1100 euro per week, and a variable transportation cost of 15 euro per hour.

In-house distribution

For the in-house scenario, the best results are obtained when using 10-pallet trucks. In the proposed solution, two of these 10-pallet trucks are used. The first truck makes a distribution pattern consisting of 11 different tours, the

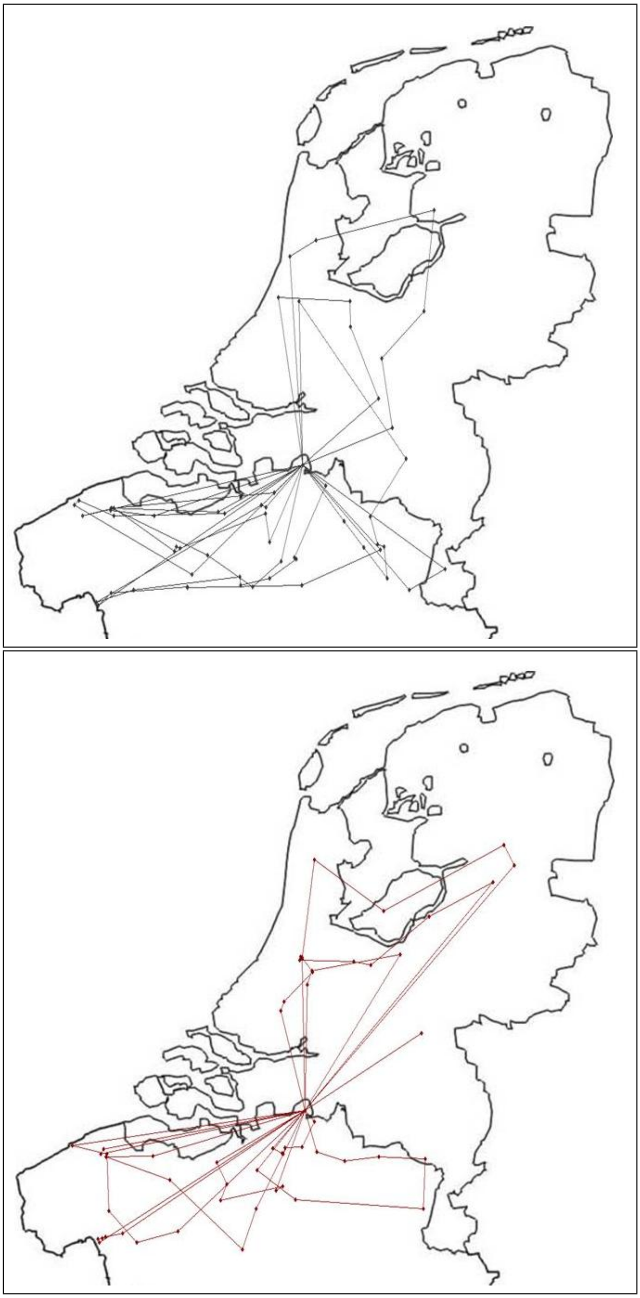


Figure 5.1: Solution for the in-house scenario

second one a distribution pattern consisting of 10 tours. The tours of these distribution patterns are shown in Figure 5.1. The characteristics of both distribution patterns are reported in Table 5.2.

Table 5.2: In-house distribution patterns

	DP1	DP2	Total
T_{min}	250h	409h	
T_{min}	40 days	60 days	
T_{max}	12 weeks	16 weeks	
Cycle time	12 weeks	16 weeks	
Transport cost rate	178.52 €/w	209.72 €/w	388.24 €/w
Delivery cost rate	141.67 €/w	188.13 €/w	329.79 €/w
Cost rate	1320.18 €/w	1397.84 €/w	2718.03 €/w
Idle time	47.9%	36.1%	
Idle days	33.3%	25.0%	
Vehicle occupation	52.1%	63.9%	
Capacity utilization	82.5%	90.7%	

Within a cycle of 12 weeks, the first vehicle is dispatched 52 times on 40 different days to make its 11 tours with differing frequencies. The second vehicle is dispatched 76 times on 60 different days to make its 10 tours within a cycle of 16 weeks. For both vehicles, some days two tours have to be made, while on other days, no tour has to be made. In its 12-week cycle, the first vehicle has 20 idle days. The second vehicle has 20 idle days in a 16-week cycle.

On the one hand, this idle time may seem an inefficiency because the (expensive) vehicles are not being used for a large portion of time. On the other hand, the fixed vehicle costs are still being accounted during this idle time and the proposed solution gives the best cost trade-off. Thus, reorganizing the distribution patterns to increase the vehicle utilization would lead to more expensive solutions.

Instead of considering the idle time of both vehicles as an inefficiency, it can be seen as an opportunity. In the cyclic solution that is proposed for customer replenishment, only customers with stable demand rates are taken into account. However, as indicated before, sometimes a number of opportunistic customers appear, ordering big volumes. To satisfy these unstable, unpredictable customer orders, the S&M office does not need a separate vehicle. The two vehicles that are already being used have enough time left to make such 'off-cycle' deliveries within their cyclic patterns. Thus, the idle time of the vehicles can be seen as the 'spare capacity' to hedge against demand and travel time uncertainties.

Outsourced distribution

For the outsourcing scenario, the best results are obtained when using the larger 15-pallet trucks. The proposed solution consists of three distribution patterns. The first distribution pattern consists of 4 different tours, the second one of 9 tours, and the third one has 5 tours. The characteristics of this solution are shown in Table 5.3.

Table 5.3: Outsourced distribution patterns

	DP1	DP2	DP3	Total
T_{min}	65h	168h	187h	
T_{min}	13 days	25 days	27 days	
T_{max}	28 weeks	6 weeks	16 weeks	
Cycle time	28 weeks	6 weeks	16 weeks	
Transport cost rate	22.57 €/w	262.85 €/w	117.20 €/w	402.62 €/w
Delivery cost rate	16.25 €/w	209.17 €/w	76.88 €/w	302.29 €/w
Cost rate	38.82 €/w	472.02 €/w	194.07 €/w	704.91 €/w
Idle time	94.2%	30.0%	70.9%	
Idle days	90.7%	16.7%	66.3%	
Vehicle occupation	5.8%	70.0%	29.1%	
Capacity utilization	53.7%	69.4%	84.1%	

Within a 28-week cycle, the first vehicle is dispatched only 13 times to make its 4 tours with differing frequencies. The second vehicle is dispatched 31 times on 25 different days to make its 9 tours within a cycle of 6 weeks. The third vehicle is dispatched 27 times every 16 weeks. This outsourcing solution has much more idle time for the vehicles. However, in this case, the fixed vehicle cost is carried by the LSP, and therefore it is the LSP's responsibility to combine the proposed routes with other routes that serve its other customers.

In the outsourcing solution, the only objective is to minimize the sum of transportation and delivery costs, whereas in the in-house solution, a trade-off with the fixed vehicle costs is envisaged. In the in-house solution, tour frequencies are aligned such that a single vehicle can cover more tours. This explains why the sum of transportation and delivery costs is higher (718,03 euro per week vs. 704,91 euro per week).

Tables 5.2 and 5.3 also illustrate the impact of the vehicle type. In the in-house solution, the vehicles are touring 46 hours per week ($= 250/12 + 409/16$), while in the outsourcing solution, the (larger) vehicles travel 42 hours per week ($= 65/28 + 168/6 + 187/16$). This means that delivery frequencies are slightly lower and delivery quantities slightly larger when using the larger vehicles. This is in line with the findings from the design of experiments in Section 4.1.

The S&M office can also use our tool to quantify the impact of the customer restrictions. E.g. by reorganizing their storage rooms, some customers may be able to make space for an extra pallet. Thus, they can decrease their replenishment frequency and allow a vehicle to make one or more extra tours. The S&M office can use the results from our tool to discuss issues like these with its customers and maybe offer a revision of its tariffs for customers that are willing to cooperate.

As an illustration, the savings heuristic was run once more with the 10-pallet vehicles, but after increasing all customer storage capacities with 50%. In the result, two vehicles are still needed, but the sum of transportation and delivery costs decreases with 14%, from 718.03 to 619.20 euro per week.

5.4 Conclusion

This chapter presents a real-life case study, in which our modelling and solution approach for cyclic inventory routing can be applied, thus illustrating the practical relevance of our work. The three main conclusions from this case study are the following.

1. Our solution approach can be used to select the appropriate vehicle type and design distribution patterns for cyclic customer replenishment.
2. Our solution approach can assist a distributor and its customers in tariff negotiations by quantifying the impact of restrictions such as limited customer storage capacities.
3. Our solution approach proves to be rather robust, i.e. the proposed solutions have an inherent slack to hedge against demand uncertainties.

Chapter 6

Conclusion

6.1 Concluding summary

In the literature on cyclic inventory routing, the objective is to find a trade-off between variable distribution and inventory costs: dispatching, stopover, transportation, ordering and holding costs. However, the cyclic inventory routing has a long-term perspective, with a time horizon of several weeks to months. Therefore, the analysis has to be extended to include decisions on the vehicle fleet size by taking fixed vehicle costs into account, instead of only variable costs. This dissertation presents the first modelling and solution approach that finds three-way cost trade-offs between (i) variable distribution costs, (ii) variable inventory costs, and (iii) fixed vehicle fleet costs, taking cyclic inventory routing closer to the practitioner's needs.

Traditional inventory routing models do not consider fixed vehicle costs, and therefore design replenishment routes without considering the assignment of these routes to vehicles. When fixed vehicle costs are included, the assignment of tours to vehicles becomes an essential part of the problem. Therefore, we have generalized the traditional routing concept of single tours to the concept of distribution patterns, which consist of a set of tours made with possibly different frequencies by a single vehicle. Furthermore, our modelling approach takes into account realistic restrictions often ignored in the literature. These include the limited customer storage capacities and the cycle time restrictions based on (i) loading, unloading and travelling times, and (ii) driving time restrictions. Table 6.1 compares the characteristics of papers found in the literature on cyclic inventory routing to our approach.

Generalizing the routing concept to distribution patterns adds to complexity considerably, because the different tours that a vehicle makes, can have different frequencies. First, a frequency combination for the tours assigned to a single vehicle has to be determined and then, a schedule has to be constructed that indicates which tour should be made when, taking into account possible driving

Table 6.1: Characteristics of different approaches in literature

Authors	Route design	Reorder policy	CCAP	VCAP	Min. cycle time	Application	Problem size
Larson [23]	single route	up-to reordering	yes	yes	-	yes	14
Webb and Larson [32]	routesets	up-to reordering	yes	yes	-	-	100
Anily and Federgruen [5, 6]	single route	zero reordering	-	yes	-	-	10000
Anily [2]	single route	zero reordering	-	yes	-	-	1000
Gallego and Simchi-Levi [19]	direct shipping	zero reordering	-	yes	-	-	-
Bramel and Simchi-Levi [9]	single route	zero reordering	-	yes	-	-	50
Viswanathan and Mathur [30]	nested routes	zero reordering	-	yes	-	-	1000
Chan et al. [11]	single route	zero reordering	-	yes	-	-	200
Qu et al. [25]	nested routes	periodic review	-	-	-	-	50
Anily and Bramel [3, 4]	single route	zero reordering	-	yes	-	-	-
Gaur and Fisher [20]	single route	zero reordering	-	yes	-	yes	207
Sindhuchao et al. [28]	single route	zero reordering	-	yes	vehicle frequency	-	50
Raa et al. [1, 27]	distribution pattern	zero reordering	yes	yes	yes	yes	120

time restrictions. In our solution approach, these complex subproblems are solved heuristically with respectively an enumeration heuristic and an insertion heuristic.

The solution approach consists of two constructive heuristics, a sequential insertion heuristic and a parallel savings heuristic, and an improvement heuristic, that are embedded in two metaheuristic frameworks: a simple multi-start framework and a more advanced column generation framework.

Computational results of our solution approach are very promising. An extensive design of experiments in which instances with varying characteristics are solved, shows that the solution approach is flexible enough to find the appropriate three-way cost trade-off under any circumstances. Our solution approach outperforms an existing insertion heuristic found in the literature for cycling inventory routing without considering fixed vehicle costs. The difference in performance is due to the use of the more general distribution pattern routing concept. Another approach in the literature finds the optimal solutions for a set of very small problem instances. The average gap between our results and the optimal solutions is only 1.23%.

After the theoretical problem instances, our solution is also applied to a real-life case. This case study proves the relevance of this dissertation for practitioners by showing how our solution approach can be used in setting up a cyclic distribution strategy. It helps in selecting the appropriate vehicle type and may be used to quantify the effect of customer restrictions. From the case study, it can also be concluded that our solution approach has a certain inherent robustness and naturally hedges against demand uncertainty somewhat.

6.2 Further extensions

The modelling and solution approach for cyclic inventory routing presented in this dissertation is highly generic. As a result, it can easily be extended to accommodate additional real-life side-constraints, such as driving time regulations, customer time windows for delivery, multiple vehicle types, multiple products, etc.

When extending the approach to multiple depots, the distribution pattern concept may need to be extended. In a distribution pattern, all tours start and end in the same depot. This should then be generalized such that a tour is merely a trip between two depots via one or more customers. This generalization will of course complicate the problem of aligning the different tours of a vehicle in the scheduling subproblem.

Another interesting extension is to bring the replenishment of the depot into scope. In our approach, it is assumed that there is always enough inventory in the depot for loading the vehicles, and the costs of inventory holding in the depot are ignored. A natural extension would thus be to take management of this central inventory into account.

The basic assumption of cyclic planning is that customer demand rates are constant. Of course, this assumption is not always valid. However, in the real-life case study, it is shown that our cyclic solution approach has some inherent robustness. Investigating how the approach can be extended to explicitly take demand uncertainty into account is an interesting avenue for future research. A preliminary study is reported in [26].

The main advantage of a cyclic approach is its predictability. All customer always know when their next replenishment will be. This reduces the nervousness and thus the customer demand variability and the safety stocks that are being maintained. Furthermore, some of the remaining demand variability is absorbed in the distribution phase because of the inherent robustness of the cyclic distribution strategy. As a result, the demand that the central depot generates at its supplier(s) shows even less variability. Thus, the cyclic approach reduces variability throughout the system. Quantifying this variability reduction is a final interesting avenue for further research.

Bibliography

[1] E. H. Aghezzaf, B. Raa, and H. Van Landeghem. Modeling inventory routing problems in supply chains of high consumption products. *European Journal of Operational Research*, 169(3):1048–1063, 2006.

[2] S. Anily. The general multiretailer eoq problem with vehicle-routing costs. *European Journal of Operational Research*, 79(3):451–473, 1994.

[3] S. Anily and J. Bramel. An asymptotic 98.5%-effective lower bound on fixed partition policies for the inventory-routing problem. *Discrete Applied Mathematics*, 145(1):22–39, 2004.

[4] S. Anily and J. Bramel. A probabilistic analysis of a fixed partition policy for the inventory-routing problem. *Naval Research Logistics*, 51(7):925–948, 2004.

[5] S. Anily and A. Federgruen. A class of euclidean routing-problems with general-route cost-functions. *Mathematics of Operations Research*, 15(2):268–285, 1990.

[6] S. Anily and A. Federgruen. 2-echelon distribution-systems with vehicle-routing costs and central inventories. *Operations Research*, 41(1):37–47, 1993.

[7] C. Barnhart, E. L. Johnson, G. L. Nemhauser, M. W. P. Savelsbergh, and P. H. Vance. Branch-and-price: Column generation for solving huge integer programs. *Operations Research*, 46(3):316–329, 1998.

[8] W.J. Bell, L.M. Dalberto, M.L. Fisher, A.J. Greenfield, R. Jaikumar, P. Kedia, R.G. Mack, and P.J. Prutzman. Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces*, 13(6):4–23, 1983.

[9] J. Bramel and D. Simchi-Levi. A location based heuristic for general routing-problems. *Operations Research*, 43(4):649–660, 1995.

[10] D. G. Cattrysse and L. N. Vanwassenhove. A survey of algorithms for the generalized assignment problem. *European Journal of Operational Research*, 60(3):260–272, 1992.

[11] L. M. A. Chan, A. Federgruen, and D. Simchi-Levi. Probabilistic analyses and practical algorithms for inventory-routing models. *Operations Research*, 46(1):96–106, 1998.

[12] G. Clarke and J.W. Wright. Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research*, 12(4):568–581, 1964.

[13] M. Dror and M. Ball. Inventory routing - reduction from an annual to a short-period problem. *Naval Research Logistics*, 34(6):891–905, 1987.

[14] M. Dror, M. Ball, and Golden B. A computational comparison of algorithms for the inventory routing problem. *Annals of Operations Research*, 4:3–23, 1985.

[15] W. Dullaert, G.K. Janssens, K. Sörensen, and B. Vernimmen. New heuristics for the fleet size and mix vehicle routing problem with time windows. *Journal of the Operational Research Society*, 53:1232–1238, 2002.

[16] Eilon, Samuel and Christofides, Nicos. The loading problem. *Management Science*, 17(5):259–268, 1971.

[17] A. Federgruen and P. Zipkin. A combined vehicle-routing and inventory allocation problem. *Operations Research*, 32(5):1019–1037, 1984.

[18] B. Fleischmann. The vehicle routing problem with multiple use of vehicles. Working Paper, Universität Hamburg, Fachbereich Wirtschaftwissenschaffen, 1990.

[19] G. Gallego and D. Simchi-Levi. On the effectiveness of direct shipping strategy for the one-warehouse multi-retailer r-systems. *Management Science*, 36(2):240–243, 1990.

[20] V. Gaur and M. L. Fisher. A periodic inventory routing problem at a supermarket chain. *Operations Research*, 52(6):813–822, 2004.

[21] J. Hahm and C. A. Yano. The economic lot and delivery scheduling problem - powers of 2 policies. *Transportation Science*, 29(3):222–241, 1995.

[22] A. J. Kleywegt, V. S. Nori, and M. W. P. Savelsbergh. The stochastic inventory routing problem with direct deliveries. *Transportation Science*, 36(1):94–118, 2002.

[23] R. C. Larson. Transporting sludge to the 106-mile site - an inventory routing model for fleet sizing and logistics system-design. *Transportation Science*, 22(3):186–198, 1988.

[24] M. E. Lubbecke and J. Desrosiers. Selected topics in column generation. *Operations Research*, 53(6):1007–1023, 2005.

[25] W. W. Qu, J. H. Bookbinder, and P. Iyogun. An integrated inventory-transportation system with modified periodic policy for multiple products. *European Journal of Operational Research*, 115(2):254–269, 1999.

[26] B. Raa and E. H. Aghezzaf. The cyclical inventory routing problem with uncertain demands and travel times. In *CD-ROM Proceedings of the 15th Mini-EURO Conference (Managing Uncertainty in Decision Support Models)*, 2004.

[27] B. Raa and E. H. Aghezzaf. Designing distribution patterns for long-term inventory-routing with constant demand rates. *International Journal of Production Economics*, to be published.

[28] S. Sindhuchao, H. E. Romeijn, E. Akcali, and R. Boondiskulchok. An integrated inventory-routing system for multi-item joint replenishment with limited vehicle capacity. *Journal of Global Optimization*, 32(1):93–118, 2005.

[29] M. M. Solomon. Algorithms for the vehicle-routing and scheduling problems with time window constraints. *Operations Research*, 35(2):254–265, 1987.

[30] S. Viswanathan and K. Mathur. Integrating routing and inventory decisions in one-warehouse multiretailer multiproduct distribution systems. *Management Science*, 43(3):294–312, 1997.

[31] M. Waller, M. E. Johnson, and T. Davis. Vendor-managed inventory in the retail supply chain. *Journal of Business Logistics*, 20(1):183–203, 1999.

[32] I. R. Webb and R. C. Larson. Period and phase of customer replenishment - a new approach to the strategic inventory/routing problem. *European Journal of Operational Research*, 85(1):132–148, 1995.

Appendix A

Computational results

This appendix gives the computational results for all instances in our design of experiments reported in Section 4.1. In this design of experiments, all instances are solved four times:

1. Using the column generation solution approach and the single tour routing concept (ST).
2. Using the column generation solution approach and the multi-tour routing concept (MT).
3. Using the column generation solution approach and the distribution pattern routing concept (DP-CG).
4. Using the multi-start solution approach and the distribution pattern routing concept (DP-MS).

Results of the first three are used to study the impact of the routing concept, while the results of the last two are used to compare the column generation and the multi-start solution approach.

A 10×2^5 Factorial Design is considered, with the following five factors: the vehicle capacity (vcap, second column), the customer storage capacity (ccap, third column), the holding cost rate (hc, fourth column), the number of customers (nr, fifth column) and the size of the service area (area, sixth column). For all combinations of factors, ten instances are generated, indicated by i in the seventh column.

The following solution characteristics are reported and used in explaining how the various cost trade-offs are obtained.

- The total cost rate (Tot) and the decomposition into its five cost components: (i) fixed vehicle costs (Veh), (ii) transportation costs (Tra), (iii) vehicle dispatching costs (Dsp), (iv) delivery costs (Del) and (v) customer holding costs (Hld).

- The calculation time (CPU).
- The number of vehicles and the number of tours in the solution (V,T).
- The utilization of the vehicle, i.e. the percentage of time it is being used (Util).
- The average number of customers per tour (C/T).

The cumulative average stock level, which is also used in the discussion of the results in Section 4.1, can be derived by dividing the customer holding cost rate (Hld) by the holding cost rate parameter (hc).

Table A.1: Results for our design of experiments

concept	vcap	ccap	hc	nr	area	i	Tot	Veh	Tra	Dsp	Del	Hld	CPU	V,T	Util	C/T
ST	100u	No	10c	[80,120]	[150,175]	0	1022.65	750	153.42	10.88	38.59	69.76	23.62	15,15	0.2529	5.73
MT	100u	No	10c	[80,120]	[150,175]	0	464.91	200	141.88	10.96	25.19	86.89	110.65	4,20	0.8171	4.30
DP-MS	100u	No	10c	[80,120]	[150,175]	0	458.99	200	129.76	10.90	21.67	96.67	359.58	4,23	0.7442	3.74
DP-CG	100u	No	10c	[80,120]	[150,175]	0	456.02	200	126.83	10.81	21.67	96.71	394.44	4,22	0.7315	3.91
ST	100u	No	10c	[80,120]	[150,175]	1	1330.11	900	265.00	16.96	63.50	84.65	78.07	18,18	0.3571	6.67
MT	100u	No	10c	[80,120]	[150,175]	1	697.94	300	217.76	16.67	37.44	126.08	377.01	6,27	0.8303	4.44
DP-MS	100u	No	10c	[80,120]	[150,175]	1	696.19	250	206.02	16.96	25.65	197.57	781.47	5,44	0.8997	2.73
DP-CG	100u	No	10c	[80,120]	[150,175]	1	696.53	300	212.25	16.54	36.98	130.76	1016.47	6,28	0.8126	4.29
ST	100u	No	10c	[80,120]	[150,175]	2	1273.44	850	254.38	17.10	71.80	80.16	225.96	17,17	0.3801	6.82
MT	100u	No	10c	[80,120]	[150,175]	2	690.47	300	217.22	18.33	41.08	113.84	246.17	6,26	0.8509	4.46
DP-MS	100u	No	10c	[80,120]	[150,175]	2	686.33	250	208.17	18.69	26.04	183.43	757.47	5,44	0.9175	2.64
DP-CG	100u	No	10c	[80,120]	[150,175]	2	687.33	300	195.01	17.92	34.11	140.29	679.98	6,32	0.7585	3.63
ST	100u	No	10c	[80,120]	[150,175]	3	950.35	650	181.54	11.44	46.02	61.35	15.04	13,13	0.3432	6.46
MT	100u	No	10c	[80,120]	[150,175]	3	456.85	200	133.60	12.50	26.83	83.92	119.99	4,20	0.8025	4.20
DP-MS	100u	No	10c	[80,120]	[150,175]	3	455.73	200	123.23	11.21	22.83	98.46	321.99	4,22	0.7262	3.82
DP-CG	100u	No	10c	[80,120]	[150,175]	3	451.40	200	123.02	11.35	24.41	92.61	371.43	4,21	0.7361	4.00
ST	100u	No	10c	[80,120]	[150,175]	4	1112.14	800	184.69	11.79	38.81	76.85	216.46	16,16	0.2715	5.88
MT	100u	No	10c	[80,120]	[150,175]	4	561.65	250	173.06	13.13	27.21	98.26	97.10	5,23	0.7785	4.09
DP-MS	100u	No	10c	[80,120]	[150,175]	4	543.11	200	168.91	13.04	19.48	141.68	354.65	4,32	0.9070	2.94
DP-CG	100u	No	10c	[80,120]	[150,175]	4	540.59	200	168.39	13.32	19.91	138.97	502.62	4,32	0.9093	2.94
ST	100u	No	10c	[80,120]	[150,175]	5	1439.87	1050	221.18	15.51	55.31	97.87	201.54	21,21	0.2598	5.62
MT	100u	No	10c	[80,120]	[150,175]	5	685.23	250	196.13	16.67	22.15	200.28	356.94	5,46	0.8479	2.57
DP-MS	100u	No	10c	[80,120]	[150,175]	5	647.03	250	197.76	15.52	29.22	154.53	863.71	5,34	0.8829	3.47
DP-CG	100u	No	10c	[80,120]	[150,175]	5	651.67	250	206.31	16.20	30.60	148.57	863.62	5,35	0.9217	3.37
ST	100u	No	10c	[80,120]	[150,175]	6	1345.34	950	230.70	15.25	58.27	91.12	226.97	19,19	0.2991	6.11
MT	100u	No	10c	[80,120]	[150,175]	6	649.45	250	193.62	16.33	25.66	163.83	351.86	5,37	0.8554	3.14
DP-MS	100u	No	10c	[80,120]	[150,175]	6	624.46	250	191.32	15.78	32.03	135.32	742.36	5,30	0.8768	3.87
DP-CG	100u	No	10c	[80,120]	[150,175]	6	625.25	250	188.31	16.07	31.69	139.18	735.70	5,31	0.8665	3.74
ST	100u	No	10c	[80,120]	[150,175]	7	1249.59	850	233.00	16.39	72.17	78.03	210.05	17,17	0.3587	6.35
MT	100u	No	10c	[80,120]	[150,175]	7	653.97	250	199.07	15.75	23.62	165.53	231.76	5,36	0.8604	3.00
DP-MS	100u	No	10c	[80,120]	[150,175]	7	605.01	250	180.83	15.73	33.23	125.22	595.93	5,28	0.8476	3.86
DP-CG	100u	No	10c	[80,120]	[150,175]	7	607.08	250	179.80	16.20	32.94	128.14	682.84	5,29	0.8450	3.72
ST	100u	No	10c	[80,120]	[150,175]	8	1229.61	850	220.40	14.86	63.37	80.98	25.98	17,17	0.3311	6.59
MT	100u	No	10c	[80,120]	[150,175]	8	630.93	250	192.88	15.08	26.79	146.18	215.30	5,32	0.8523	3.50
DP-MS	100u	No	10c	[80,120]	[150,175]	8	605.69	250	182.13	14.58	32.03	126.95	694.41	5,27	0.8402	4.15
DP-CG	100u	No	10c	[80,120]	[150,175]	8	603.17	250	178.79	14.83	31.97	127.57	820.39	5,28	0.8300	4.00
ST	100u	No	10c	[80,120]	[150,175]	9	1518.86	1100	249.78	15.40	56.00	97.69	20.28	22,22	0.2704	4.68
MT	100u	No	10c	[80,120]	[150,175]	9	675.86	250	199.33	15.36	18.58	192.59	251.34	5,43	0.8341	2.40
DP-MS	100u	No	10c	[80,120]	[150,175]	9	649.21	250	209.40	15.40	25.85	148.56	590.05	5,35	0.9043	2.94
DP-CG	100u	No	10c	[80,120]	[150,175]	9	659.17	250	211.23	15.52	23.53	158.89	506.65	5,36	0.8994	2.86
ST	100u	No	10c	[80,120]	[75,100]	0	684.25	400	162.67	12.71	73.02	35.86	18.69	8,8	0.6068	10.38
MT	100u	No	10c	[80,120]	[75,100]	0	404.05	150	112.65	12.50	20.75	108.15	101.11	3,25	0.9029	3.32
DP-MS	100u	No	10c	[80,120]	[75,100]	0	378.40	150	103.13	12.08	27.78	85.41	347.34	3,20	0.9051	4.15
DP-CG	100u	No	10c	[80,120]	[75,100]	0	379.19	150	101.48	12.08	25.52	90.10	303.39	3,21	0.8772	3.95
ST	100u	No	10c	[80,120]	[75,100]	1	831.79	500	181.10	15.00	88.75	46.94	47.37	10,10	0.5612	11.00
MT	100u	No	10c	[80,120]	[75,100]	1	500.77	200	123.18	14.79	27.71	135.09	193.96	4,29	0.7789	3.79
DP-MS	100u	No	10c	[80,120]	[75,100]	1	474.23	200	122.12	15.00	40.21	96.90	793.48	4,21	0.8539	5.24
DP-CG	100u	No	10c	[80,120]	[75,100]	1	473.05	200	117.94	14.58	36.96	103.57	877.18	4,22	0.8136	5.00
ST	100u	No	10c	[80,120]	[75,100]	2	819.76	450	191.41	18.00	120.38	39.97	41.24	9,9	0.7388	13.00
MT	100u	No	10c	[80,120]	[75,100]	2	503.59	200	117.87	16.50	33.19	136.03	177.74	4,29	0.8017	4.03
DP-MS	100u	No	10c	[80,120]	[75,100]	2	480.91	200	109.76	17.19	41.30	112.66	941.52	4,25	0.8229	4.68
DP-CG	100u	No	10c	[80,120]	[75,100]	2	482.31	200	115.25	16.88	44.17	106.02	1064.89	4,23	0.8617	5.09
ST	100u	No	10c	[80,120]	[75,100]	3	577.98	350	117.40	10.63	66.56	33.39	11.97	7,7	0.5552	11.57
MT	100u	No	10c	[80,120]	[75,100]	3	336.19	150	82.02	10.42	35.73	58.03	138.19	3,12	0.8402	6.75
DP-MS	100u	No	10c	[80,120]	[75,100]	3	332.17	150	73.65	11.25	29.58	67.68	319.11	3,15	0.7495	5.40
DP-CG	100u	No	10c	[80,120]	[75,100]	3	333.16	150	75.10	10.79	29.13	68.15	336.32	3,15	0.7498	5.40

ST	100u	No	10c	[80,120]	[75,100]	4	750.20	450	167.44	13.21	78.58	40.97	20.85	9,9	0.5651	10.33
MT	100u	No	10c	[80,120]	[75,100]	4	444.63	200	115.15	12.92	33.33	83.23	102.23	4,18	0.7688	5.17
DP-MS	100u	No	10c	[80,120]	[75,100]	4	423.37	150	113.78	13.19	22.63	123.77	477.24	3,28	0.9306	3.32
DP-CG	100u	No	10c	[80,120]	[75,100]	4	437.50	200	107.65	12.60	33.18	84.07	570.80	4,18	0.7347	5.17
ST	100u	No	10c	[80,120]	[75,100]	5	803.58	450	178.45	17.08	117.50	40.55	56.85	9,9	0.7043	13.22
MT	100u	No	10c	[80,120]	[75,100]	5	482.06	200	106.20	16.13	35.06	124.67	368.03	4,27	0.7624	4.41
DP-MS	100u	No	10c	[80,120]	[75,100]	5	464.12	200	101.34	15.83	44.06	102.89	1032.02	4,22	0.7966	5.41
DP-CG	100u	No	10c	[80,120]	[75,100]	5	463.66	200	102.13	15.83	43.65	102.05	1301.62	4,22	0.7973	5.41
ST	100u	No	10c	[80,120]	[75,100]	6	707.97	400	163.78	14.25	93.88	36.07	23.11	8,8	0.6791	12.25
MT	100u	No	10c	[80,120]	[75,100]	6	419.48	150	104.88	14.00	24.50	126.10	157.50	3,28	0.9035	3.50
DP-MS	100u	No	10c	[80,120]	[75,100]	6	398.24	150	102.45	13.96	32.55	99.27	615.82	3,24	0.9568	4.08
DP-CG	100u	No	10c	[80,120]	[75,100]	6	409.09	150	102.41	14.33	28.48	113.87	764.75	3,26	0.9257	3.77
ST	100u	No	10c	[80,120]	[75,100]	7	828.52	500	183.50	14.38	82.19	48.46	53.12	10,10	0.5472	10.80
MT	100u	No	10c	[80,120]	[75,100]	7	495.69	200	122.89	14.88	28.81	129.11	208.92	4,28	0.7851	3.86
DP-MS	100u	No	10c	[80,120]	[75,100]	7	469.65	200	116.22	15.00	38.23	100.20	731.32	4,22	0.8169	4.91
DP-CG	100u	No	10c	[80,120]	[75,100]	7	470.33	200	121.91	15.00	40.83	92.59	636.51	4,20	0.8569	5.40
ST	100u	No	10c	[80,120]	[75,100]	8	668.53	400	151.43	12.08	67.08	37.93	21.62	8,8	0.5629	10.63
MT	100u	No	10c	[80,120]	[75,100]	8	419.55	200	97.81	12.29	30.42	79.03	108.40	4,17	0.6745	5.00
DP-MS	100u	No	10c	[80,120]	[75,100]	8	384.36	150	103.62	13.33	26.72	90.69	375.34	3,21	0.9094	4.05
DP-CG	100u	No	10c	[80,120]	[75,100]	8	382.54	150	98.15	12.71	25.94	95.75	454.91	3,22	0.8673	3.86
ST	100u	No	10c	[80,120]	[75,100]	9	640.52	350	163.11	13.13	86.25	28.04	10.20	7,7	0.7433	12.29
MT	100u	No	10c	[80,120]	[75,100]	9	406.92	200	96.33	12.08	37.08	61.43	101.12	4,14	0.7087	6.14
DP-MS	100u	No	10c	[80,120]	[75,100]	9	356.01	150	90.59	11.25	30.42	73.76	394.53	3,16	0.8505	5.38
DP-CG	100u	No	10c	[80,120]	[75,100]	9	357.13	150	88.08	11.46	28.75	78.84	436.05	3,18	0.8244	4.78
ST	100u	No	10c	[30,70]	[150,175]	0	851.32	650	114.18	7.20	19.43	60.52	2.99	13,13	0.1976	4.38
MT	100u	No	10c	[30,70]	[150,175]	0	352.89	150	107.66	8.06	15.69	71.48	50.72	3,17	0.7960	3.35
DP-MS	100u	No	10c	[30,70]	[150,175]	0	349.28	150	102.75	8.46	15.33	72.74	100.93	3,18	0.7691	3.17
DP-CG	100u	No	10c	[30,70]	[150,175]	0	347.04	150	103.00	7.75	15.21	71.07	136.60	3,17	0.7636	3.35
ST	100u	No	10c	[30,70]	[150,175]	1	464.46	350	64.16	4.28	12.20	33.81	0.69	7,7	0.2116	4.71
MT	100u	No	10c	[30,70]	[150,175]	1	213.06	100	58.22	5.00	8.13	41.72	9.35	2,10	0.6492	3.30
DP-MS	100u	No	10c	[30,70]	[150,175]	1	207.68	100	55.59	4.51	9.44	38.13	20.98	2,9	0.6377	3.67
DP-CG	100u	No	10c	[30,70]	[150,175]	1	208.34	100	53.43	4.48	8.75	41.68	18.13	2,9	0.6106	3.67
ST	100u	No	10c	[30,70]	[150,175]	2	864.62	650	124.41	7.57	19.42	63.23	58.78	13,13	0.2114	4.77
MT	100u	No	10c	[30,70]	[150,175]	2	378.10	150	119.29	7.92	12.97	87.92	44.07	3,19	0.8368	3.26
DP-MS	100u	No	10c	[30,70]	[150,175]	2	366.72	150	115.19	8.02	15.47	78.05	112.02	3,17	0.8357	3.65
DP-CG	100u	No	10c	[30,70]	[150,175]	2	368.66	150	116.30	8.15	15.88	78.33	125.26	3,17	0.8464	3.65
ST	100u	No	10c	[30,70]	[150,175]	3	614.80	450	101.87	5.67	15.97	41.29	0.54	9,9	0.2488	4.22
MT	100u	No	10c	[30,70]	[150,175]	3	261.39	100	81.00	6.50	7.72	66.17	15.51	2,17	0.8527	2.24
DP-MS	100u	No	10c	[30,70]	[150,175]	3	243.29	100	70.86	5.94	9.61	56.88	31.94	2,14	0.7848	2.71
DP-CG	100u	No	10c	[30,70]	[150,175]	3	242.89	100	74.86	6.04	10.31	51.68	47.57	2,14	0.8283	2.71
ST	100u	No	10c	[30,70]	[150,175]	4	693.06	500	118.90	7.40	19.74	47.02	1.47	10,10	0.2660	4.70
MT	100u	No	10c	[30,70]	[150,175]	4	344.63	150	107.38	7.92	13.33	66.00	17.07	3,15	0.7736	3.13
DP-MS	100u	No	10c	[30,70]	[150,175]	4	336.69	150	97.97	7.76	13.10	67.86	55.07	3,16	0.7181	2.94
DP-CG	100u	No	10c	[30,70]	[150,175]	4	335.95	150	98.27	8.16	13.09	66.43	51.52	3,16	0.7230	2.94
ST	100u	No	10c	[30,70]	[150,175]	5	716.67	500	139.16	8.17	23.13	46.21	2.01	10,10	0.3102	5.10
MT	100u	No	10c	[30,70]	[150,175]	5	357.27	150	104.89	8.88	12.41	81.10	24.04	3,19	0.7601	2.68
DP-MS	100u	No	10c	[30,70]	[150,175]	5	348.91	150	97.83	8.56	12.69	79.83	60.95	3,18	0.7206	2.83
DP-CG	100u	No	10c	[30,70]	[150,175]	5	351.52	150	104.51	8.61	14.38	74.02	74.48	3,17	0.7722	3.00
ST	100u	No	10c	[30,70]	[150,175]	6	547.97	400	88.89	5.21	15.83	38.04	1.28	8,8	0.2509	4.88
MT	100u	No	10c	[30,70]	[150,175]	6	248.09	100	75.82	6.50	8.68	57.09	6.83	2,15	0.8216	2.60
DP-MS	100u	No	10c	[30,70]	[150,175]	6	242.28	100	71.74	5.65	9.38	55.51	33.41	2,13	0.7857	3.00
DP-CG	100u	No	10c	[30,70]	[150,175]	6	240.21	100	74.11	5.63	10.89	49.59	39.69	2,12	0.8240	3.25
ST	100u	No	10c	[30,70]	[150,175]	7	406.32	300	64.74	3.71	9.46	28.41	0.51	6,6	0.2347	5.17
MT	100u	No	10c	[30,70]	[150,175]	7	206.59	100	53.68	4.14	6.68	42.08	7.07	2,10	0.5826	3.10
DP-MS	100u	No	10c	[30,70]	[150,175]	7	203.92	100	53.73	3.88	7.44	38.88	18.95	2,9	0.5891	3.44
DP-CG	100u	No	10c	[30,70]	[150,175]	7	204.37	100	52.96	4.06	7.28	40.06	28.89	2,10	0.5831	3.10
ST	100u	No	10c	[30,70]	[150,175]	8	550.50	400	90.68	5.44	15.56	38.81	1.32	8,8	0.2546	5.38
MT	100u	No	10c	[30,70]	[150,175]	8	253.60	100	80.33	5.92	9.71	57.64	15.92	2,13	0.8648	3.31
DP-MS	100u	No	10c	[30,70]	[150,175]	8	252.15	100	80.87	5.81	11.39	54.08	42.64	2,13	0.8889	3.31
DP-CG	100u	No	10c	[30,70]	[150,175]	8	253.51	100	79.86	5.94	10.19	57.53	61.72	2,13	0.8671	3.31
ST	100u	No	10c	[30,70]	[150,175]	9	642.07	450	117.45	7.73	25.83	41.06	1.54	9,9	0.3107	5.22
MT	100u	No	10c	[30,70]	[150,175]	9	288.49	100	87.11	7.80	9.26	84.33	36.66	2,20	0.9391	2.35
DP-MS	100u	No	10c	[30,70]	[150,175]	9	317.79	150	89.05	7.40	13.49	57.85	56.99	3,14	0.6688	3.36
DP-CG	100u	No	10c	[30,70]	[150,175]	9	318.79	150	81.02	7.35	11.52	68.91	68.19	3,16	0.6073	2.94
ST	100u	No	10c	[30,70]	[75,100]	0	296.75	200	56.51	4.17	16.67	19.40	0.79	4,4	0.3657	7.75
MT	100u	No	10c	[30,70]	[75,100]	0	187.44	100	38.23	4.50	8.56	36.15	2.24	2,8	0.4818	3.88
DP-MS	100u	No	10c	[30,70]	[75,100]	0	184.82	100	37.94	4.58	10.00	32.30	22.67	2,7	0.4984	4.43
DP-CG	100u	No	10c	[30,70]	[75,100]	0	141.38	50	36.35	4.58	8.13	42.33	19.43	1,10	0.9235	3.10
ST	100u	No	10c	[30,70]	[75,100]	1	395.47	250	77.86	6.61	36.43	24.57	10.13	5,5	0.4747	10.20
MT	100u	No	10c	[30,70]	[75,100]	1	225.22	100	54.21	7.29	18.54	45.18	13.08	2,10	0.7746	5.10
DP-MS	100u	No	10c	[30,70]	[75,100]	1	221.99	100	51.30	7.08	18.13	45.48	82.15	2,10	0.7426	5.10
DP-CG	100u	No	10c	[30,70]	[75,100]	1	221.89	100	48.89	7.50	17.92	47.58	161.58	2,11	0.7251	4.64
ST	100u	No	10c	[30,70]	[75,100]	2	373.18	250	70.18	5.21	23.33	24.46	2.83	5,5	0.3766	8.60
MT	100u	No	10c	[30,70]	[75,100]	2	207.76	100	48.21	5.83	14.08	39.63	15.73	2,9	0.6507	4.78
DP-MS	100u	No	10c	[30,70]	[75,100]	2	204.15	100	45.44	6.04	14.58	38.09	45.34	2,9	0.6365	4.78
DP-CG	100u	No	10c	[30,70]	[75,100]	2	205.63	100	48.13	5.42	15.00	37.08	52.12	2,8	0.6563	5.38
ST	100u	No	10c	[30,70]	[75,100]	3	456.79	300	85.40	7.29	35.52	28.58	3.89	6,6	0.4156	8.67
MT	100u	No	10c	[30,70]	[75,100]	3	264.59	100	74.30	7.71	13.33	69.24	20.73	2,16	0.8822	3.25
DP-MS	100u	No	10c	[30,70]	[75,100]	3	243.92	100	66.97	8.44	18.18	50.34	92.61	2,13	0.8908	4.00
DP-CG	100u	No	10c	[30,70]	[75,100]	3	242.82	100	67.94	7.50	18.33	49.05	100.99	2,12	0.8891	4.33
ST	100u	No	10c	[30,70]	[75,100]	4	376.23	250	71.10	5.63	25.63	23.88	5.71	5,5	0.3933	9.00
MT	100u	No	10c	[30,70]	[75,100]	4	211.28	100	49.07	5.83	14.58	41.79	24.11	2,9	0.6642	5.00
DP-MS	100u	No	10c	[30,70]	[75,100]	4	211.38	100	47.56	6.15	14.11	43.56	53.52	2,10	0.6496	4.50
DP-CG	100u	No	10c	[30,70]	[75,100]	4	210.80	100	49.04	5.83	15.00	40.92	103.29	2,9	0.6691	5.00
ST	100u	No	10c	[30,70]	[75,100]	5	543.13	300	140.19	11.25	65.63	26.07	5.93	6,6	0.7097	11.17
MT	100u	No	10c	[30,70]	[75,100]	5	334.29	150	88.62	10.00	29.58	56.08	63.16	3,12	0.8222	5.58
DP-MS	100u	No	10c	[30,70]	[75,100]	5	328.94	150	80.48	10.83	26.67	60.96	177.53	3,14	0.7596	4.79
DP-CG	100u	No	10c	[30,70]	[75,100]	5	329.53	150	80.96	10.21	25.63	62.73	182.79	3,14	0.7484	4.79

ST	100u	No	10c	[30,70]	[75,100]	6	462.91	300	86.53	7.38	40.38	28.63	8.43	6,6	0.4393	10.33
MT	100u	No	10c	[30,70]	[75,100]	6	248.74	100	60.47	8.38	17.25	62.64	54.71	2,15	0.8242	4.13
DP-MS	100u	No	10c	[30,70]	[75,100]	6	238.07	100	54.57	8.02	18.18	57.30	175.47	2,13	0.7822	4.77
DP-CG	100u	No	10c	[30,70]	[75,100]	6	239.43	100	55.08	7.97	18.65	57.73	133.56	2,13	0.7917	4.77
ST	100u	No	10c	[30,70]	[75,100]	7	278.42	200	42.53	3.54	13.02	19.33	1.03	4,4	0.2807	7.50
MT	100u	No	10c	[30,70]	[75,100]	7	129.60	50	34.00	4.00	7.50	34.10	2.91	1,8	0.8541	3.75
DP-MS	100u	No	10c	[30,70]	[75,100]	7	125.93	50	32.54	3.75	8.28	31.36	22.89	1,7	0.8431	4.29
DP-CG	100u	No	10c	[30,70]	[75,100]	7	125.72	50	32.05	3.75	8.13	31.80	28.74	1,7	0.8310	4.29
ST	100u	No	10c	[30,70]	[75,100]	8	362.06	200	87.95	7.50	48.75	17.86	3.89	4,4	0.7180	12.50
MT	100u	No	10c	[30,70]	[75,100]	8	227.19	100	52.07	7.50	15.63	52.00	21.38	2,12	0.7230	4.17
DP-MS	100u	No	10c	[30,70]	[75,100]	8	217.42	100	43.68	7.08	15.73	50.93	69.00	2,11	0.6492	4.55
DP-CG	100u	No	10c	[30,70]	[75,100]	8	218.04	100	46.94	7.08	18.33	45.68	107.33	2,10	0.7089	5.00
ST	100u	No	10c	[30,70]	[75,100]	9	469.42	300	100.89	7.08	32.71	28.74	5.58	6,6	0.4461	9.17
MT	100u	No	10c	[30,70]	[75,100]	9	269.91	100	75.09	7.67	12.54	74.62	36.39	2,17	0.8783	3.24
DP-MS	100u	No	10c	[30,70]	[75,100]	9	257.28	100	71.06	7.50	15.00	63.72	97.52	2,14	0.8734	3.93
DP-CG	100u	No	10c	[30,70]	[75,100]	9	255.84	100	72.98	8.44	16.88	57.54	87.49	2,14	0.9246	3.93
ST	100u	No	1c	[80,120]	[150,175]	0	965.03	750	159.73	10.26	37.88	7.15	29.07	15,15	0.2577	5.73
MT	100u	No	1c	[80,120]	[150,175]	0	294.06	150	100.88	10.56	8.27	24.35	693.02	3,53	0.7173	1.62
DP-MS	100u	No	1c	[80,120]	[150,175]	0	290.92	150	97.17	10.33	8.26	25.16	4897.79	3,54	0.6948	1.59
DP-CG	100u	No	1c	[80,120]	[150,175]	0	290.37	150	96.93	10.39	8.46	24.59	4103.43	3,53	0.6956	1.62
ST	100u	No	1c	[80,120]	[150,175]	1	1338.31	1000	253.31	16.64	58.84	9.53	231.90	20,20	0.3054	6.00
MT	100u	No	1c	[80,120]	[150,175]	1	485.65	250	169.76	17.14	13.56	35.19	1805.43	5,78	0.7194	1.54
DP-MS	100u	No	1c	[80,120]	[150,175]	1	434.69	200	166.98	17.05	12.13	38.52	15210.60	4,85	0.8781	1.41
DP-CG	100u	No	1c	[80,120]	[150,175]	1	479.01	250	161.11	16.51	12.72	38.68	10139.10	5,82	0.6831	1.46
ST	100u	No	1c	[80,120]	[150,175]	2	1282.40	950	241.70	17.29	64.34	9.07	229.02	19,19	0.3194	6.11
MT	100u	No	1c	[80,120]	[150,175]	2	427.40	200	158.13	17.70	10.92	40.65	754.54	4,91	0.8378	1.27
DP-MS	100u	No	1c	[80,120]	[150,175]	2	421.88	200	153.13	17.16	11.54	40.06	16592.70	4,88	0.8174	1.32
DP-CG	100u	No	1c	[80,120]	[150,175]	2	421.30	200	152.60	17.16	11.81	39.74	13215.70	4,86	0.8169	1.35
ST	100u	No	1c	[80,120]	[150,175]	3	925.76	700	163.41	11.19	44.58	6.58	78.00	14,14	0.2941	6.00
MT	100u	No	1c	[80,120]	[150,175]	3	296.71	150	101.08	11.52	8.30	25.81	344.35	3,57	0.7267	1.47
DP-MS	100u	No	1c	[80,120]	[150,175]	3	291.05	150	95.52	11.19	9.32	25.03	5601.24	3,54	0.7015	1.56
DP-CG	100u	No	1c	[80,120]	[150,175]	3	291.01	150	95.85	11.30	9.32	24.54	3097.33	3,53	0.7043	1.58
ST	100u	No	1c	[80,120]	[150,175]	4	1048.09	800	189.06	11.96	39.40	7.67	215.07	16,16	0.2772	5.88
MT	100u	No	1c	[80,120]	[150,175]	4	383.65	200	132.32	12.34	8.58	30.41	449.63	4,66	0.6821	1.42
DP-MS	100u	No	1c	[80,120]	[150,175]	4	378.59	200	125.95	12.11	8.67	31.85	6299.13	4,68	0.6547	1.38
DP-CG	100u	No	1c	[80,120]	[150,175]	4	378.02	200	126.65	12.00	9.13	30.24	6217.08	4,64	0.6598	1.47
ST	100u	No	1c	[80,120]	[150,175]	5	1418.51	1100	240.60	15.22	52.13	10.56	233.73	22,22	0.2588	5.36
MT	100u	No	1c	[80,120]	[150,175]	5	425.48	200	162.59	16.00	11.86	35.03	1288.79	4,77	0.8516	1.53
DP-MS	100u	No	1c	[80,120]	[150,175]	5	417.51	200	154.48	15.43	12.55	35.05	14932.50	4,75	0.8185	1.57
DP-CG	100u	No	1c	[80,120]	[150,175]	5	419.22	200	154.26	15.47	11.49	38.01	8966.68	4,81	0.8112	1.46
ST	100u	No	1c	[80,120]	[150,175]	6	1238.82	900	253.10	15.79	61.54	8.39	214.03	18,18	0.3418	6.44
MT	100u	No	1c	[80,120]	[150,175]	6	415.26	200	152.24	16.01	11.75	35.26	935.24	4,78	0.8078	1.49
DP-MS	100u	No	1c	[80,120]	[150,175]	6	410.02	200	147.32	15.54	12.90	34.25	12839.70	4,74	0.7916	1.57
DP-CG	100u	No	1c	[80,120]	[150,175]	6	410.06	200	146.62	15.32	12.12	36.00	9442.63	4,76	0.7824	1.53
ST	100u	No	1c	[80,120]	[150,175]	7	1158.28	850	223.32	15.90	61.13	7.94	222.73	17,17	0.3322	6.35
MT	100u	No	1c	[80,120]	[150,175]	7	405.49	200	143.16	15.73	10.36	36.24	643.51	4,79	0.7596	1.37
DP-MS	100u	No	1c	[80,120]	[150,175]	7	399.63	200	137.31	15.21	11.07	36.04	15896.50	4,76	0.7364	1.42
DP-CG	100u	No	1c	[80,120]	[150,175]	7	399.92	200	137.53	15.20	11.19	36.00	9740.00	4,76	0.7380	1.42
ST	100u	No	1c	[80,120]	[150,175]	8	1247.79	950	220.64	14.98	53.09	9.08	219.34	19,19	0.2831	5.89
MT	100u	No	1c	[80,120]	[150,175]	8	403.57	200	142.78	14.99	10.96	34.83	1060.80	4,76	0.7571	1.47
DP-MS	100u	No	1c	[80,120]	[150,175]	8	395.51	200	135.18	14.51	12.28	33.54	13356.70	4,71	0.7307	1.58
DP-CG	100u	No	1c	[80,120]	[150,175]	8	396.19	200	136.39	14.57	11.81	33.43	8789.46	4,71	0.7331	1.58
ST	100u	No	1c	[80,120]	[150,175]	9	1437.87	1150	223.95	14.36	38.56	11.00	89.56	23,23	0.2198	4.48
MT	100u	No	1c	[80,120]	[150,175]	9	427.69	200	167.20	15.68	9.76	35.05	765.13	4,81	0.8557	1.27
DP-MS	100u	No	1c	[80,120]	[150,175]	9	416.95	200	157.86	14.51	10.37	34.20	8666.09	4,73	0.8133	1.41
DP-CG	100u	No	1c	[80,120]	[150,175]	9	417.53	200	159.19	14.62	11.25	32.48	6493.42	4,70	0.8250	1.47
ST	100u	No	1c	[80,120]	[75,100]	0	640.59	400	158.24	12.50	66.25	3.60	17.24	8,8	0.5758	10.38
MT	100u	No	1c	[80,120]	[75,100]	0	227.19	100	81.30	12.44	9.13	24.31	1064.16	2,56	0.9472	1.48
DP-MS	100u	No	1c	[80,120]	[75,100]	0	224.97	100	79.43	12.08	10.21	23.25	9256.81	2,52	0.9406	1.60
DP-CG	100u	No	1c	[80,120]	[75,100]	0	222.16	100	77.24	11.95	10.39	22.58	11498.70	2,50	0.9229	1.66
ST	100u	No	1c	[80,120]	[75,100]	1	803.62	500	189.43	15.92	93.75	4.52	51.87	10,10	0.5899	11.00
MT	100u	No	1c	[80,120]	[75,100]	1	298.42	150	88.60	15.26	11.06	33.50	2835.81	3,74	0.7115	1.49
DP-MS	100u	No	1c	[80,120]	[75,100]	1	293.49	150	85.49	14.50	13.85	29.65	15900.40	3,63	0.7112	1.75
DP-CG	100u	No	1c	[80,120]	[75,100]	1	293.75	150	86.51	14.52	13.50	29.22	15515.60	3,62	0.7141	1.77
ST	100u	No	1c	[80,120]	[75,100]	2	780.64	450	188.25	17.71	120.63	4.06	93.39	9,9	0.7329	13.00
MT	100u	No	1c	[80,120]	[75,100]	2	301.88	150	87.08	17.14	12.80	34.85	2030.63	3,77	0.7333	1.52
DP-MS	100u	No	1c	[80,120]	[75,100]	2	296.86	150	83.74	16.69	14.70	31.73	20612.60	3,68	0.7268	1.72
DP-CG	100u	No	1c	[80,120]	[75,100]	2	296.42	150	84.71	16.58	15.56	29.58	17533.00	3,64	0.7384	1.83
ST	100u	No	1c	[80,120]	[75,100]	3	582.01	400	108.21	10.83	59.17	3.80	15.14	8,8	0.4442	10.13
MT	100u	No	1c	[80,120]	[75,100]	3	196.33	100	53.64	11.32	8.80	22.57	754.85	2,51	0.6986	1.59
DP-MS	100u	No	1c	[80,120]	[75,100]	3	194.14	100	52.45	10.96	10.10	20.63	6228.61	2,45	0.7004	1.80
DP-CG	100u	No	1c	[80,120]	[75,100]	3	193.82	100	52.27	11.01	10.00	20.53	3840.89	2,45	0.6982	1.80
ST	100u	No	1c	[80,120]	[75,100]	4	708.48	450	166.60	13.07	74.58	4.23	26.86	9,9	0.5520	10.33
MT	100u	No	1c	[80,120]	[75,100]	4	280.53	150	81.68	13.25	12.17	23.43	1024.26	3,52	0.6656	1.79
DP-MS	100u	No	1c	[80,120]	[75,100]	4	278.57	150	79.16	12.76	11.25	25.40	10933.90	3,55	0.6398	1.69
DP-CG	100u	No	1c	[80,120]	[75,100]	4	278.07	150	78.19	12.59	10.95	26.34	8584.07	3,56	0.6306	1.66
ST	100u	No	1c	[80,120]	[75,100]	5	796.56	500	173.12	16.13	102.60	4.71	99.87	10,10	0.5853	11.90
MT	100u	No	1c	[80,120]	[75,100]	5	291.75	150	78.76	16.54	13.54	32.90	3312.95	3,73	0.6883	1.63
DP-MS	100u	No	1c	[80,120]	[75,100]	5	284.86	150	73.82	15.96	15.76	29.32	23299.40	3,63	0.6744	1.89
DP-CG	100u	No	1c	[80,120]	[75,100]	5	284.74	150	73.92	16.01	15.41	29.41	20399.60	3,63	0.6724	1.89
ST	100u	No	1c	[80,120]	[75,100]	6	694.42	400	184.06	15.00	91.88	3.48	31.16	8,8	0.7174	12.25
MT	100u	No	1c	[80,120]	[75,100]	6	227.55	100	75.03	14.04	11.13	27.35	2006.28	2,61	0.9398	1.61
DP-MS	100u	No	1c	[80,120]	[75,100]	6	225.00	100	73.00	13.63	11.85	26.53	14484.50	2,58	0.9268	1.69
DP-CG	100u	No	1c	[80,120]	[75,100]	6	223.94	100	72.17	13.59	12.38	25.81	16358.70	2,56	0.9259	1.75
ST	100u	No	1c	[80,120]	[75,100]	7	787.36	500	178.35	15.17	89.17	4.67	70.28	10,10	0.5581	10.80
MT	100u	No	1c	[80,120]	[75,100]	7	301.37	150	93.77	14.80	11.89	30.91	1026.84	3,67	0.7434	1.61
DP-MS	100u	No	1c	[80,120]	[75,100]	7	295.65	150	88.60	14.30	12.63	30.12	16016.20	3,63	0.7167	1.71
DP-CG	100u	No	1c	[80,120]	[75,100]	7	296.58	150	90.55	14.64	13.43	27.95	14100.30	3,60	0.7370	1.80

ST	100u	No	1c	[80,120]	[75,100]	8	660.47	400	171.66	13.33	71.88	3.60	16.75	8,8	0.6239	10.63
MT	100u	No	1c	[80,120]	[75,100]	8	224.79	100	76.74	13.04	9.01	26.00	1149.42	2,60	0.9151	1.42
DP-MS	100u	No	1c	[80,120]	[75,100]	8	223.44	100	75.93	12.36	10.32	24.83	10023.40	2,54	0.9163	1.57
DP-CG	100u	No	1c	[80,120]	[75,100]	8	222.86	100	75.74	12.49	9.48	25.14	9903.68	2,56	0.9058	1.52
ST	100u	No	1c	[80,120]	[75,100]	9	617.68	400	138.92	11.25	63.75	3.76	28.84	8,8	0.5238	10.75
MT	100u	No	1c	[80,120]	[75,100]	9	212.48	100	68.57	11.38	9.71	22.83	822.20	2,50	0.8350	1.72
DP-MS	100u	No	1c	[80,120]	[75,100]	9	209.51	100	66.21	11.23	10.37	21.71	7836.12	2,47	0.8217	1.83
DP-CG	100u	No	1c	[80,120]	[75,100]	9	209.26	100	65.79	11.13	10.94	21.41	5779.98	2,46	0.8240	1.87
ST	100u	No	1c	[30,70]	[150,175]	0	798.74	650	114.59	7.11	20.77	6.27	5.64	13,13	0.2005	4.38
MT	100u	No	1c	[30,70]	[150,175]	0	220.31	100	90.45	8.12	6.51	15.23	165.86	2,36	0.9366	1.58
DP-MS	100u	No	1c	[30,70]	[150,175]	0	210.18	100	79.52	7.48	5.82	17.36	1290.58	2,38	0.8290	1.50
DP-CG	100u	No	1c	[30,70]	[150,175]	0	209.53	100	77.92	7.59	5.31	18.72	756.83	2,42	0.8105	1.36
ST	100u	No	1c	[30,70]	[150,175]	1	435.06	350	65.38	4.28	12.04	3.37	0.55	7,7	0.2139	4.71
MT	100u	No	1c	[30,70]	[150,175]	1	163.44	100	45.68	4.84	3.66	9.26	17.38	2,22	0.4869	1.50
DP-MS	100u	No	1c	[30,70]	[150,175]	1	159.63	100	42.50	4.39	4.53	8.21	179.20	2,18	0.4657	1.83
DP-CG	100u	No	1c	[30,70]	[150,175]	1	159.66	100	42.00	4.44	4.19	9.02	169.32	2,20	0.4579	1.65
ST	100u	No	1c	[30,70]	[150,175]	2	806.61	650	123.07	7.58	19.69	6.27	52.91	13,13	0.2102	4.77
MT	100u	No	1c	[30,70]	[150,175]	2	274.88	150	92.08	8.21	5.99	18.59	169.44	3,42	0.6299	1.48
DP-MS	100u	No	1c	[30,70]	[150,175]	2	270.11	150	87.99	7.80	6.45	17.87	1518.82	3,38	0.6076	1.63
DP-CG	100u	No	1c	[30,70]	[150,175]	2	269.50	150	86.35	7.92	5.68	19.56	1435.23	3,43	0.5930	1.44
ST	100u	No	1c	[30,70]	[150,175]	3	605.44	500	82.58	5.31	12.65	4.90	0.85	10,10	0.1825	3.80
MT	100u	No	1c	[30,70]	[150,175]	3	183.15	100	60.85	5.88	4.09	12.33	28.46	2,28	0.6317	1.36
DP-MS	100u	No	1c	[30,70]	[150,175]	3	176.85	100	54.11	5.77	4.15	12.83	265.83	2,29	0.5748	1.31
DP-CG	100u	No	1c	[30,70]	[150,175]	3	176.12	100	53.30	5.65	4.07	13.10	204.83	2,29	0.5657	1.31
ST	100u	No	1c	[30,70]	[150,175]	4	649.13	500	116.45	7.23	20.64	4.81	2.77	10,10	0.2637	4.70
MT	100u	No	1c	[30,70]	[150,175]	4	210.66	100	82.83	8.00	5.88	13.96	58.51	2,32	0.8637	1.47
DP-MS	100u	No	1c	[30,70]	[150,175]	4	206.46	100	77.23	7.73	5.22	16.28	664.22	2,36	0.8055	1.31
DP-CG	100u	No	1c	[30,70]	[150,175]	4	204.99	100	76.49	7.42	5.36	15.70	528.47	2,34	0.7973	1.38
ST	100u	No	1c	[30,70]	[150,175]	5	713.01	550	123.35	8.07	26.43	5.17	2.37	11,11	0.2653	4.64
MT	100u	No	1c	[30,70]	[150,175]	5	218.68	100	87.65	8.54	5.73	16.76	122.88	2,38	0.9088	1.34
DP-MS	100u	No	1c	[30,70]	[150,175]	5	216.45	100	84.84	8.61	5.82	17.17	1114.69	2,39	0.8874	1.31
DP-CG	100u	No	1c	[30,70]	[150,175]	5	216.42	100	84.54	8.68	5.79	17.41	1337.74	2,40	0.8853	1.28
ST	100u	No	1c	[30,70]	[150,175]	6	551.31	450	79.35	5.18	12.44	4.34	1.25	9,9	0.1959	4.33
MT	100u	No	1c	[30,70]	[150,175]	6	179.91	100	58.52	5.75	4.09	11.55	28.99	2,27	0.6107	1.44
DP-MS	100u	No	1c	[30,70]	[150,175]	6	175.49	100	52.36	5.40	3.56	14.17	598.89	2,31	0.5483	1.26
DP-CG	100u	No	1c	[30,70]	[150,175]	6	175.73	100	52.87	5.42	3.80	13.65	414.81	2,30	0.5557	1.30
ST	100u	No	1c	[30,70]	[150,175]	7	379.39	300	63.60	3.63	9.25	2.91	0.49	6,6	0.2303	5.17
MT	100u	No	1c	[30,70]	[150,175]	7	159.03	100	43.41	4.05	3.44	8.12	11.76	2,19	0.4554	1.63
DP-MS	100u	No	1c	[30,70]	[150,175]	7	107.83	50	41.84	4.03	3.16	8.80	283.17	1,21	0.8771	1.48
DP-CG	100u	No	1c	[30,70]	[150,175]	7	108.30	50	41.16	3.96	2.76	10.42	236.11	1,24	0.8540	1.29
ST	100u	No	1c	[30,70]	[150,175]	8	522.35	400	96.02	5.55	16.97	3.82	2.79	8,8	0.2704	5.38
MT	100u	No	1c	[30,70]	[150,175]	8	187.06	100	64.30	6.14	4.59	12.03	32.57	2,28	0.6700	1.54
DP-MS	100u	No	1c	[30,70]	[150,175]	8	180.81	100	57.42	5.67	4.34	13.38	449.61	2,29	0.6036	1.48
DP-CG	100u	No	1c	[30,70]	[150,175]	8	180.87	100	57.56	5.67	4.27	13.37	310.09	2,29	0.6039	1.48
ST	100u	No	1c	[30,70]	[150,175]	9	631.14	500	101.52	6.90	17.96	4.75	1.56	10,10	0.2314	4.70
MT	100u	No	1c	[30,70]	[150,175]	9	197.37	100	69.67	7.54	5.04	15.13	52.36	2,35	0.7378	1.34
DP-MS	100u	No	1c	[30,70]	[150,175]	9	192.75	100	64.95	7.14	5.17	15.49	954.48	2,34	0.6950	1.38
DP-CG	100u	No	1c	[30,70]	[150,175]	9	192.35	100	64.24	7.19	5.02	15.90	811.93	2,35	0.6880	1.34
ST	100u	No	1c	[30,70]	[75,100]	0	279.57	200	57.21	4.17	16.25	1.95	0.96	4,4	0.3660	7.75
MT	100u	No	1c	[30,70]	[75,100]	0	94.80	50	27.81	4.55	3.52	8.92	4.91	1,20	0.6653	1.55
DP-MS	100u	No	1c	[30,70]	[75,100]	0	93.68	50	26.37	4.50	3.58	9.22	261.15	1,21	0.6416	1.48
DP-CG	100u	No	1c	[30,70]	[75,100]	0	93.76	50	26.50	4.58	3.40	9.28	139.89	1,21	0.6413	1.48
ST	100u	No	1c	[30,70]	[75,100]	1	379.56	250	84.19	6.67	36.25	2.45	3.59	5,5	0.4952	10.20
MT	100u	No	1c	[30,70]	[75,100]	1	115.02	50	38.11	7.50	6.38	13.03	112.57	1,30	0.9821	1.70
DP-MS	100u	No	1c	[30,70]	[75,100]	1	113.46	50	36.33	7.19	6.13	13.82	1864.73	1,31	0.9384	1.65
DP-CG	100u	No	1c	[30,70]	[75,100]	1	113.68	50	36.65	7.19	5.97	13.88	1834.61	1,31	0.9397	1.65
ST	100u	No	1c	[30,70]	[75,100]	2	347.26	250	65.52	5.42	23.96	2.37	2.99	5,5	0.3653	8.60
MT	100u	No	1c	[30,70]	[75,100]	2	107.61	50	36.27	5.75	5.38	10.21	27.44	1,23	0.8826	1.87
DP-MS	100u	No	1c	[30,70]	[75,100]	2	105.03	50	33.28	5.75	5.09	10.91	1289.14	1,25	0.8257	1.72
DP-CG	100u	No	1c	[30,70]	[75,100]	2	105.01	50	32.81	5.81	4.91	11.49	753.39	1,26	0.8147	1.65
ST	100u	No	1c	[30,70]	[75,100]	3	436.57	300	89.58	7.44	36.70	2.85	5.81	6,6	0.4327	8.67
MT	100u	No	1c	[30,70]	[75,100]	3	177.46	100	50.20	7.50	7.19	12.57	301.98	2,28	0.6019	1.86
DP-MS	100u	No	1c	[30,70]	[75,100]	3	173.79	100	46.21	7.34	6.91	13.33	1256.68	2,29	0.5632	1.79
DP-CG	100u	No	1c	[30,70]	[75,100]	3	173.49	100	44.97	7.27	6.23	15.02	1451.65	2,33	0.5435	1.58
ST	100u	No	1c	[30,70]	[75,100]	4	349.04	250	66.36	5.50	24.75	2.42	2.65	5,5	0.3725	9.00
MT	100u	No	1c	[30,70]	[75,100]	4	109.30	50	37.23	5.75	5.63	10.69	64.20	1,23	0.9049	1.96
DP-MS	100u	No	1c	[30,70]	[75,100]	4	108.20	50	35.59	5.75	5.47	11.39	1408.85	1,25	0.8737	1.80
DP-CG	100u	No	1c	[30,70]	[75,100]	4	107.83	50	35.78	5.73	6.15	10.18	838.58	1,22	0.8932	2.05
ST	100u	No	1c	[30,70]	[75,100]	5	551.53	350	133.94	10.42	53.96	3.22	10.37	7,7	0.5488	9.57
MT	100u	No	1c	[30,70]	[75,100]	5	199.52	100	60.96	10.42	7.56	20.58	630.84	2,46	0.7327	1.46
DP-MS	100u	No	1c	[30,70]	[75,100]	5	195.42	100	57.40	9.99	8.84	19.18	3445.78	2,41	0.7138	1.63
DP-CG	100u	No	1c	[30,70]	[75,100]	5	195.50	100	57.75	9.92	8.64	19.20	2970.61	2,41	0.7132	1.63
ST	100u	No	1c	[30,70]	[75,100]	6	442.61	300	90.12	7.71	41.98	2.80	5.21	6,6	0.4574	10.33
MT	100u	No	1c	[30,70]	[75,100]	6	170.76	100	40.83	7.79	7.32	14.82	1017.60	2,33	0.5291	1.88
DP-MS	100u	No	1c	[30,70]	[75,100]	6	167.29	100	36.78	7.33	7.12	16.07	2883.94	2,34	0.4871	1.82
DP-CG	100u	No	1c	[30,70]	[75,100]	6	167.55	100	37.94	7.43	7.68	14.50	3719.98	2,31	0.5050	2.00
ST	100u	No	1c	[30,70]	[75,100]	7	261.72	200	42.61	3.54	13.65	1.92	0.98	4,4	0.2850	7.50
MT	100u	No	1c	[30,70]	[75,100]	7	92.02	50	27.45	4.00	3.75	6.82	6.82	1,16	0.6513	1.88
DP-MS	100u	No	1c	[30,70]	[75,100]	7	88.97	50	24.22	3.75	3.26	7.74	191.20	1,17	0.5790	1.76
DP-CG	100u	No	1c	[30,70]	[75,100]	7	88.87	50	24.42	3.68	3.37	7.40	52.01	1,16	0.5833	1.88
ST	100u	No	1c	[30,70]	[75,100]	8	367.66	250	72.10	6.75	36.38	2.44	3.99	5,5	0.4559	10.00
MT	100u	No	1c	[30,70]	[75,100]	8	113.67	50	37.42	7.00	6.25	13.00	43.66	1,28	0.9549	1.79
DP-MS	100u	No	1c	[30,70]	[75,100]	8	111.37	50	35.17	7.00	7.31	11.89	1548.59	1,26	0.9440	1.92
DP-CG	100u	No	1c	[30,70]	[75,100]	8	111.36	50	34.79	7.13	7.19	12.26	1073.43	1,27	0.9377	1.85
ST	100u	No	1c	[30,70]	[75,100]	9	483.75	350	95.93	6.88	27.50	3.44	10.31	7,7	0.3512	7.86
MT	100u	No	1c	[30,70]	[75,100]	9	182.49	100	53.29	7.76	5.90	15.54	232.34	2,36	0.6149	1.53
DP-MS	100u	No	1c	[30,70]	[75,100]	9	178.64	100	49.62	7.22	5.91	15.90	2199.23	2,34	0.5775	1.62
DP-CG	100u	No	1c	[30,70]	[75,100]	9	178.42	100	50.11	7.18	6.55	14.58	2245.81	2,31	0.5891	1.77

ST	100u	Yes	10c	[80,120]	[150,175]	0	1132.54	800	219.89	13.38	38.69	60.58	95.44	16,16 0.3104	5.38
MT	100u	Yes	10c	[80,120]	[150,175]	0	630.86	300	217.45	14.17	35.38	63.87	58.32	6,18 0.8104	4.78
DP-MS	100u	Yes	10c	[80,120]	[150,175]	0	636.86	300	222.01	13.96	33.92	66.97	122.77	6,19 0.8162	4.53
DP-CG	100u	Yes	10c	[80,120]	[150,175]	0	634.00	300	216.97	13.83	33.21	69.99	141.24	6,19 0.7987	4.53
ST	100u	Yes	10c	[80,120]	[150,175]	1	1443.16	950	321.51	20.25	75.90	75.51	214.28	19,19 0.4085	6.32
MT	100u	Yes	10c	[80,120]	[150,175]	1	865.72	400	291.86	19.58	50.04	104.24	215.01	8,25 0.8256	4.80
DP-MS	100u	Yes	10c	[80,120]	[150,175]	1	857.64	400	288.09	18.09	51.08	100.38	379.13	8,23 0.8163	5.22
DP-CG	100u	Yes	10c	[80,120]	[150,175]	1	864.00	400	294.48	18.23	51.20	100.09	641.45	8,23 0.8305	5.22
ST	100u	Yes	10c	[80,120]	[150,175]	2	1421.27	950	305.84	19.54	66.73	79.16	234.60	19,19 0.3818	6.11
MT	100u	Yes	10c	[80,120]	[150,175]	2	854.12	400	285.71	19.50	52.94	95.97	160.59	8,24 0.8216	4.83
DP-MS	100u	Yes	10c	[80,120]	[150,175]	2	813.06	350	281.60	20.58	50.99	109.89	330.00	7,28 0.9261	4.14
DP-CG	100u	Yes	10c	[80,120]	[150,175]	2	854.17	400	285.32	19.25	51.31	98.28	495.70	8,24 0.8149	4.83
ST	100u	Yes	10c	[80,120]	[150,175]	3	1010.77	700	195.66	12.58	41.73	60.80	86.15	14,14 0.3299	6.00
MT	100u	Yes	10c	[80,120]	[150,175]	3	613.51	300	195.49	13.56	39.56	64.89	74.95	6,16 0.7644	5.25
DP-MS	100u	Yes	10c	[80,120]	[150,175]	3	563.37	250	192.13	14.06	36.63	70.55	121.46	5,18 0.8939	4.67
DP-CG	100u	Yes	10c	[80,120]	[150,175]	3	567.34	250	196.66	14.38	37.50	68.80	102.53	5,18 0.9149	4.67
ST	100u	Yes	10c	[80,120]	[150,175]	4	1227.02	850	250.89	15.27	43.82	67.03	211.19	17,17 0.3329	5.53
MT	100u	Yes	10c	[80,120]	[150,175]	4	732.44	350	254.85	16.67	43.69	67.24	58.04	7,19 0.8223	4.95
DP-MS	100u	Yes	10c	[80,120]	[150,175]	4	702.17	300	267.88	16.25	36.25	81.79	159.39	6,22 0.9629	4.27
DP-CG	100u	Yes	10c	[80,120]	[150,175]	4	737.92	350	257.55	15.25	37.71	77.41	194.01	7,20 0.8023	4.70
ST	100u	Yes	10c	[80,120]	[150,175]	5	1567.00	1100	303.12	19.44	58.08	86.37	229.89	22,22 0.3177	5.36
MT	100u	Yes	10c	[80,120]	[150,175]	5	855.91	400	291.78	19.38	49.42	95.34	217.39	8,25 0.8228	4.72
DP-MS	100u	Yes	10c	[80,120]	[150,175]	5	869.45	400	298.10	19.85	49.32	102.18	338.97	8,27 0.8372	4.37
DP-CG	100u	Yes	10c	[80,120]	[150,175]	5	876.42	400	312.23	19.06	48.80	96.32	377.22	8,26 0.8626	4.54
ST	100u	Yes	10c	[80,120]	[150,175]	6	1375.00	900	311.60	19.21	72.54	71.65	203.96	18,18 0.4159	6.44
MT	100u	Yes	10c	[80,120]	[150,175]	6	779.86	350	268.64	18.75	48.38	94.10	184.99	7,24 0.8793	4.83
DP-MS	100u	Yes	10c	[80,120]	[150,175]	6	773.78	350	260.09	18.33	46.35	99.00	344.43	7,24 0.8503	4.83
DP-CG	100u	Yes	10c	[80,120]	[150,175]	6	771.12	350	259.49	17.29	45.83	98.50	504.42	7,23 0.8433	5.04
ST	100u	Yes	10c	[80,120]	[150,175]	7	1299.48	850	292.81	18.96	70.42	67.30	214.10	17,17 0.4185	6.35
MT	100u	Yes	10c	[80,120]	[150,175]	7	768.32	350	266.88	17.21	49.75	84.48	178.63	7,20 0.8746	5.40
DP-MS	100u	Yes	10c	[80,120]	[150,175]	7	768.28	350	259.46	17.46	46.50	94.86	267.86	7,23 0.8462	4.70
DP-CG	100u	Yes	10c	[80,120]	[150,175]	7	769.02	350	265.50	19.27	49.90	84.35	263.71	7,22 0.8792	4.91
ST	100u	Yes	10c	[80,120]	[150,175]	8	1397.30	950	299.64	18.48	58.59	70.59	240.49	19,19 0.3643	5.89
MT	100u	Yes	10c	[80,120]	[150,175]	8	788.98	350	284.08	18.87	47.78	88.24	155.65	7,23 0.9144	4.87
DP-MS	100u	Yes	10c	[80,120]	[150,175]	8	771.95	350	270.82	18.42	49.23	83.49	289.81	7,22 0.8864	5.09
DP-CG	100u	Yes	10c	[80,120]	[150,175]	8	780.30	350	276.87	18.75	49.27	85.41	306.70	7,23 0.9021	4.87
ST	100u	Yes	10c	[80,120]	[150,175]	9	1604.24	1150	301.28	17.67	44.44	90.85	187.90	23,23 0.2858	4.48
MT	100u	Yes	10c	[80,120]	[150,175]	9	856.18	400	288.82	18.75	37.50	111.11	225.03	8,29 0.7775	3.55
DP-MS	100u	Yes	10c	[80,120]	[150,175]	9	807.42	350	295.44	18.75	38.82	104.40	256.48	7,29 0.9091	3.55
DP-CG	100u	Yes	10c	[80,120]	[150,175]	9	815.56	350	300.19	17.81	38.80	108.75	338.60	7,28 0.9169	3.68
ST	100u	Yes	10c	[80,120]	[75,100]	0	710.25	450	148.80	11.25	56.56	43.63	34.15	9,9 0.4639	9.22
MT	100u	Yes	10c	[80,120]	[75,100]	0	438.12	200	125.48	12.50	40.10	60.04	83.36	4,14 0.8516	5.93
DP-MS	100u	Yes	10c	[80,120]	[75,100]	0	431.93	200	115.27	13.33	36.88	66.45	145.57	4,16 0.7941	5.19
DP-CG	100u	Yes	10c	[80,120]	[75,100]	0	431.46	200	122.43	12.08	44.27	52.68	169.59	4,12 0.8623	6.92
ST	100u	Yes	10c	[80,120]	[75,100]	1	881.52	550	185.65	15.21	80.00	50.66	121.22	11,11 0.4977	10.00
MT	100u	Yes	10c	[80,120]	[75,100]	1	544.31	250	151.74	14.38	56.46	71.74	179.86	5,15 0.8600	7.33
DP-MS	100u	Yes	10c	[80,120]	[75,100]	1	549.55	250	154.15	15.00	53.65	76.75	346.68	5,17 0.8571	6.47
DP-CG	100u	Yes	10c	[80,120]	[75,100]	1	546.70	250	151.32	15.83	50.52	79.03	467.94	5,18 0.8362	6.11
ST	100u	Yes	10c	[80,120]	[75,100]	2	846.41	500	184.31	16.25	97.50	48.35	87.76	10,10 0.5916	11.70
MT	100u	Yes	10c	[80,120]	[75,100]	2	544.58	250	136.73	17.50	54.90	85.46	131.98	5,19 0.8177	6.16
DP-MS	100u	Yes	10c	[80,120]	[75,100]	2	545.12	250	132.78	17.50	51.15	93.69	559.94	5,22 0.7858	5.32
DP-CG	100u	Yes	10c	[80,120]	[75,100]	2	540.48	250	131.16	16.67	53.44	89.21	752.52	5,19 0.7877	6.16
ST	100u	Yes	10c	[80,120]	[75,100]	3	591.69	350	130.39	10.83	67.50	32.97	19.72	7,7 0.5902	11.57
MT	100u	Yes	10c	[80,120]	[75,100]	3	389.03	200	81.71	10.46	34.94	61.93	43.69	4,13 0.6242	6.23
DP-MS	100u	Yes	10c	[80,120]	[75,100]	3	340.20	150	82.12	10.94	34.43	62.71	179.50	3,14 0.8343	5.79
DP-CG	100u	Yes	10c	[80,120]	[75,100]	3	342.79	150	82.54	11.88	32.92	65.46	183.27	3,16 0.8318	5.06
ST	100u	Yes	10c	[80,120]	[75,100]	4	786.76	500	165.36	13.00	63.13	45.28	69.44	10,10 0.4659	9.30
MT	100u	Yes	10c	[80,120]	[75,100]	4	498.34	250	125.96	12.50	43.23	66.65	169.77	5,14 0.6985	6.64
DP-MS	100u	Yes	10c	[80,120]	[75,100]	4	465.74	200	135.32	13.44	39.95	77.04	219.81	4,18 0.8975	5.17
DP-CG	100u	Yes	10c	[80,120]	[75,100]	4	459.12	200	128.73	14.17	39.27	76.95	213.28	4,18 0.8704	5.17
ST	100u	Yes	10c	[80,120]	[75,100]	5	837.63	500	168.92	16.46	106.25	46.00	118.90	10,10 0.5883	11.90
MT	100u	Yes	10c	[80,120]	[75,100]	5	526.61	250	123.16	16.67	54.58	82.20	130.23	5,18 0.7668	6.61
DP-MS	100u	Yes	10c	[80,120]	[75,100]	5	525.34	250	118.25	17.71	54.79	84.59	513.47	5,20 0.7567	5.95
DP-CG	100u	Yes	10c	[80,120]	[75,100]	5	484.66	200	122.17	17.92	51.25	93.32	647.23	4,23 0.9413	5.17
ST	100u	Yes	10c	[80,120]	[75,100]	6	758.79	450	174.74	13.75	79.38	40.93	236.49	9,9 0.5823	10.89
MT	100u	Yes	10c	[80,120]	[75,100]	6	452.72	200	118.44	13.08	44.58	76.61	119.63	4,16 0.8539	6.13
DP-MS	100u	Yes	10c	[80,120]	[75,100]	6	455.35	200	119.82	13.33	43.85	78.34	264.39	4,17 0.8567	5.76
DP-CG	100u	Yes	10c	[80,120]	[75,100]	6	450.31	200	119.40	14.79	47.08	69.04	291.31	4,16 0.8842	6.13
ST	100u	Yes	10c	[80,120]	[75,100]	7	888.28	550	193.60	15.21	78.54	50.93	182.41	11,11 0.5064	9.82
MT	100u	Yes	10c	[80,120]	[75,100]	7	538.96	250	150.83	14.58	57.71	65.84	107.95	5,14 0.8642	7.71
DP-MS	100u	Yes	10c	[80,120]	[75,100]	7	546.24	250	152.10	16.04	51.15	76.96	310.00	5,18 0.8429	6.00
DP-CG	100u	Yes	10c	[80,120]	[75,100]	7	539.66	250	148.78	14.58	50.42	75.88	314.37	5,16 0.8209	6.75
ST	100u	Yes	10c	[80,120]	[75,100]	8	720.87	450	153.16	12.29	63.23	42.19	29.75	9,9 0.4934	9.44
MT	100u	Yes	10c	[80,120]	[75,100]	8	440.47	200	127.74	11.67	42.92	58.15	67.95	4,12 0.8734	7.08
DP-MS	100u	Yes	10c	[80,120]	[75,100]	8	442.93	200	125.44	13.09	41.18	63.22	154.95	4,16 0.8619	5.31
DP-CG	100u	Yes	10c	[80,120]	[75,100]	8	437.19	200	117.90	12.29	38.65	68.36	147.39	4,15 0.8096	5.67
ST	100u	Yes	10c	[80,120]	[75,100]	9	652.70	400	139.45	11.67	65.00	36.59	14.23	8,8 0.5301	10.75
MT	100u	Yes	10c	[80,120]	[75,100]	9	413.77	200	102.89	11.04	38.75	61.09	107.98	4,13 0.7399	6.62
DP-MS	100u	Yes	10c	[80,120]	[75,100]	9	416.61	200	106.70	10.83	36.98	62.10	168.95	4,13 0.7434	6.62
DP-CG	100u	Yes	10c	[80,120]	[75,100]	9	416.62	200	107.30	11.25	39.38	58.70	215.09	4,13 0.7635	6.62
ST	100u	Yes	10c	[30,70]	[150,175]	0	892.15	650	161.34	9.40	23.74	47.67	3.65	13,13 0.2706	4.38
MT	100u	Yes	10c	[30,70]	[150,175]	0	504.48	250	172.98	10.94	24.25	46.31	24.25	5,15 0.7525	3.80
DP-MS	100u	Yes	10c	[30,70]	[150,175]	0	448.41	200	166.35	9.69	22.60	49.77	39.42	4,14 0.8950	4.07
DP-CG	100u	Yes	10c	[30,70]	[150,175]	0	455.31	200	168.77	10.10	22.55	53.88	34.43	4,16 0.9073	3.56
ST	100u	Yes	10c	[30,70]	[150,175]	1	503.00	350	106.24	6.11	16.53	24.12	0.72	7,7 0.3338	4.71
MT	100u	Yes	10c	[30,70]	[150,175]	1	303.90	150	102.68	6.25	13.13	31.84	2.80	3,9 0.7319	3.67
DP-MS	100u	Yes	10c	[30,70]	[150,175]	1	299.73	150	99.52	6.04	15.10	29.06	7.60	3,8 0.7291	4.13
DP-CG	100u	Yes	10c	[30,70]	[150,175]	1	300.88	150	99.50	6.25	13.33	31.79	9.12	3,9 0.7160	3.67

ST	100u	Yes	10c	[30,70]	[150,175]	2	900.73	650	165.70	9.67	26.23	49.14	7.23	13,13	0.2815	4.77
MT	100u	Yes	10c	[30,70]	[150,175]	2	509.24	250	175.52	10.25	26.48	46.99	31.21	5,13	0.7687	4.77
DP-MS	100u	Yes	10c	[30,70]	[150,175]	2	508.36	250	168.57	10.11	25.33	54.35	48.67	5,15	0.7391	4.13
DP-CG	100u	Yes	10c	[30,70]	[150,175]	2	465.72	200	174.67	10.00	25.21	55.85	49.83	4,15	0.9478	4.13
ST	100u	Yes	10c	[30,70]	[150,175]	3	702.97	500	146.03	8.75	17.71	30.48	1.22	10,10	0.3095	3.80
MT	100u	Yes	10c	[30,70]	[150,175]	3	421.97	200	161.45	9.76	16.43	34.33	3.24	4,12	0.8364	3.17
DP-MS	100u	Yes	10c	[30,70]	[150,175]	3	405.27	200	144.48	9.69	15.99	35.12	12.02	4,12	0.7625	3.17
DP-CG	100u	Yes	10c	[30,70]	[150,175]	3	404.00	200	144.46	8.54	16.61	34.39	11.95	4,11	0.7591	3.45
ST	100u	Yes	10c	[30,70]	[150,175]	4	745.34	500	169.90	9.96	28.33	37.15	3.65	10,10	0.3789	4.70
MT	100u	Yes	10c	[30,70]	[150,175]	4	442.12	200	168.98	10.00	21.56	41.58	9.91	4,12	0.9013	3.92
DP-MS	100u	Yes	10c	[30,70]	[150,175]	4	433.11	200	154.81	9.75	20.17	48.39	19.84	4,13	0.8320	3.62
DP-CG	100u	Yes	10c	[30,70]	[150,175]	4	438.52	200	158.26	10.03	19.68	50.55	21.40	4,14	0.8451	3.36
ST	100u	Yes	10c	[30,70]	[150,175]	5	780.84	550	152.17	9.25	22.83	46.59	2.50	11,11	0.3035	4.64
MT	100u	Yes	10c	[30,70]	[150,175]	5	443.76	200	161.23	10.00	22.63	49.90	8.93	4,13	0.8757	3.92
DP-MS	100u	Yes	10c	[30,70]	[150,175]	5	436.36	200	156.22	10.52	22.45	47.17	25.83	4,13	0.8570	3.92
DP-CG	100u	Yes	10c	[30,70]	[150,175]	5	436.76	200	156.77	10.42	22.40	47.18	24.68	4,13	0.8583	3.92
ST	100u	Yes	10c	[30,70]	[150,175]	6	574.10	400	117.78	6.73	18.94	30.65	1.32	8,8	0.3256	4.88
MT	100u	Yes	10c	[30,70]	[150,175]	6	305.87	150	98.98	6.07	15.94	34.88	5.18	3,9	0.7333	4.33
DP-MS	100u	Yes	10c	[30,70]	[150,175]	6	306.49	150	97.69	6.77	14.58	37.45	14.43	3,10	0.7207	3.90
DP-CG	100u	Yes	10c	[30,70]	[150,175]	6	305.27	150	97.30	6.25	15.26	36.46	14.30	3,10	0.7198	3.90
ST	100u	Yes	10c	[30,70]	[150,175]	7	426.39	300	83.87	4.75	12.94	24.83	0.40	6,6	0.3067	5.17
MT	100u	Yes	10c	[30,70]	[150,175]	7	275.57	150	81.46	5.13	11.50	27.48	1.99	3,7	0.5911	4.43
DP-MS	100u	Yes	10c	[30,70]	[150,175]	7	274.29	150	79.35	5.38	12.31	27.25	7.07	3,8	0.5882	3.88
DP-CG	100u	Yes	10c	[30,70]	[150,175]	7	277.27	150	83.06	5.49	12.43	26.29	9.82	3,8	0.6107	3.88
ST	100u	Yes	10c	[30,70]	[150,175]	8	639.36	450	129.79	7.92	19.06	32.59	3.03	9,9	0.3153	4.78
MT	100u	Yes	10c	[30,70]	[150,175]	8	395.77	200	136.61	8.75	19.17	31.25	8.15	4,10	0.7437	4.30
DP-MS	100u	Yes	10c	[30,70]	[150,175]	8	394.36	200	134.80	8.30	19.46	31.80	14.65	4,10	0.7352	4.30
DP-CG	100u	Yes	10c	[30,70]	[150,175]	8	391.98	200	132.16	8.58	19.33	31.90	26.53	4,10	0.7251	4.30
ST	100u	Yes	10c	[30,70]	[150,175]	9	706.96	500	136.83	8.40	22.48	39.26	1.83	10,10	0.3052	4.70
MT	100u	Yes	10c	[30,70]	[150,175]	9	416.14	200	144.41	9.17	21.29	41.27	8.14	4,11	0.7921	4.27
DP-MS	100u	Yes	10c	[30,70]	[150,175]	9	415.07	200	143.06	9.31	21.17	41.53	19.83	4,12	0.7866	3.92
DP-CG	100u	Yes	10c	[30,70]	[150,175]	9	414.86	200	143.37	8.96	21.77	40.76	35.05	4,11	0.7894	4.27
ST	100u	Yes	10c	[30,70]	[75,100]	0	296.01	200	55.45	4.38	17.50	18.68	0.77	4,4	0.3678	7.75
MT	100u	Yes	10c	[30,70]	[75,100]	0	195.56	100	52.01	5.00	14.38	24.18	3.21	2,6	0.6756	5.17
DP-MS	100u	Yes	10c	[30,70]	[75,100]	0	195.48	100	52.51	4.38	15.00	23.60	6.63	2,5	0.6798	6.20
DP-CG	100u	Yes	10c	[30,70]	[75,100]	0	195.48	100	52.51	4.38	15.00	23.60	5.34	2,5	0.6798	6.20
ST	100u	Yes	10c	[30,70]	[75,100]	1	404.08	250	85.41	7.08	38.33	23.26	3.85	5,5	0.5118	10.20
MT	100u	Yes	10c	[30,70]	[75,100]	1	237.42	100	65.33	8.75	21.88	41.46	14.85	2,11	0.9272	4.64
DP-MS	100u	Yes	10c	[30,70]	[75,100]	1	234.64	100	64.87	7.92	21.67	40.18	37.98	2,10	0.9104	5.10
DP-CG	100u	Yes	10c	[30,70]	[75,100]	1	228.31	100	58.39	6.88	20.63	42.42	48.54	2,9	0.8303	5.67
ST	100u	Yes	10c	[30,70]	[75,100]	2	371.22	250	68.65	5.63	25.00	21.94	2.72	5,5	0.3820	8.60
MT	100u	Yes	10c	[30,70]	[75,100]	2	213.76	100	57.86	5.63	19.38	30.90	5.96	2,7	0.7947	6.14
DP-MS	100u	Yes	10c	[30,70]	[75,100]	2	214.54	100	56.84	5.94	18.44	33.32	20.78	2,8	0.7784	5.38
DP-CG	100u	Yes	10c	[30,70]	[75,100]	2	212.61	100	57.02	5.63	19.06	30.90	28.73	2,7	0.7838	6.14
ST	100u	Yes	10c	[30,70]	[75,100]	3	477.36	300	104.81	7.58	37.33	27.63	3.53	6,6	0.4783	8.67
MT	100u	Yes	10c	[30,70]	[75,100]	3	309.01	150	88.47	7.71	23.44	39.39	10.23	3,9	0.7511	5.78
DP-MS	100u	Yes	10c	[30,70]	[75,100]	3	303.49	150	83.87	7.43	24.41	37.78	33.98	3,9	0.7313	5.78
DP-CG	100u	Yes	10c	[30,70]	[75,100]	3	304.06	150	83.77	7.57	25.03	37.69	53.05	3,9	0.7371	5.78
ST	100u	Yes	10c	[30,70]	[75,100]	4	372.22	250	66.94	5.83	26.46	22.99	3.22	5,5	0.3846	9.00
MT	100u	Yes	10c	[30,70]	[75,100]	4	224.76	100	65.70	6.25	19.69	33.12	15.28	2,8	0.8717	5.63
DP-MS	100u	Yes	10c	[30,70]	[75,100]	4	221.78	100	62.95	7.50	22.50	28.83	26.39	2,8	0.8996	5.63
DP-CG	100u	Yes	10c	[30,70]	[75,100]	4	220.70	100	60.71	5.94	18.91	35.14	30.45	2,8	0.8165	5.63
ST	100u	Yes	10c	[30,70]	[75,100]	5	570.67	350	127.81	10.00	49.38	33.48	17.40	7,7	0.5164	9.57
MT	100u	Yes	10c	[30,70]	[75,100]	5	388.92	200	97.00	10.08	35.63	46.21	45.54	4,10	0.6898	6.70
DP-MS	100u	Yes	10c	[30,70]	[75,100]	5	350.52	150	99.62	11.25	31.41	58.24	81.98	3,14	0.9089	4.79
DP-CG	100u	Yes	10c	[30,70]	[75,100]	5	346.57	150	99.71	10.21	31.67	54.99	73.88	3,12	0.9029	5.58
ST	100u	Yes	10c	[30,70]	[75,100]	6	466.12	300	87.28	7.50	42.81	28.53	5.97	6,6	0.4521	10.33
MT	100u	Yes	10c	[30,70]	[75,100]	6	300.43	150	74.49	8.13	29.38	38.44	21.14	3,9	0.7263	6.89
DP-MS	100u	Yes	10c	[30,70]	[75,100]	6	300.54	150	76.63	7.92	30.73	35.26	62.56	3,8	0.7478	7.75
DP-CG	100u	Yes	10c	[30,70]	[75,100]	6	300.28	150	74.35	7.71	28.13	40.10	57.55	3,9	0.7116	6.89
ST	100u	Yes	10c	[30,70]	[75,100]	7	241.26	150	55.12	3.75	18.75	13.64	0.89	3,3	0.4937	10.00
MT	100u	Yes	10c	[30,70]	[75,100]	7	181.50	100	44.08	5.00	14.58	17.84	4.24	2,5	0.6121	6.00
DP-MS	100u	Yes	10c	[30,70]	[75,100]	7	179.52	100	41.98	4.79	13.85	18.89	7.35	2,5	0.5829	6.00
DP-CG	100u	Yes	10c	[30,70]	[75,100]	7	179.52	100	41.98	4.79	13.85	18.89	6.64	2,5	0.5829	6.00
ST	100u	Yes	10c	[30,70]	[75,100]	8	398.61	250	80.01	6.88	37.81	23.92	5.96	5,5	0.4901	10.00
MT	100u	Yes	10c	[30,70]	[75,100]	8	277.28	150	60.93	6.88	26.88	32.60	18.44	3,7	0.6197	7.14
DP-MS	100u	Yes	10c	[30,70]	[75,100]	8	235.14	100	65.34	8.44	23.28	38.08	34.96	2,10	0.9410	5.00
DP-CG	100u	Yes	10c	[30,70]	[75,100]	8	233.47	100	63.85	7.92	23.54	38.16	58.73	2,9	0.9253	5.56
ST	100u	Yes	10c	[30,70]	[75,100]	9	512.76	350	93.92	7.17	28.79	32.88	19.36	7,7	0.3520	7.86
MT	100u	Yes	10c	[30,70]	[75,100]	9	316.26	150	94.69	7.50	27.81	36.26	13.13	3,8	0.8203	6.88
DP-MS	100u	Yes	10c	[30,70]	[75,100]	9	312.92	150	88.94	7.29	24.90	41.80	39.12	3,9	0.7623	6.11
DP-CG	100u	Yes	10c	[30,70]	[75,100]	9	310.73	150	87.66	7.50	25.00	40.57	40.89	3,9	0.7578	6.11
ST	100u	Yes	1c	[80,120]	[150,175]	0	1081.96	800	225.66	13.29	36.83	6.18	129.01	16,16	0.3134	5.38
MT	100u	Yes	1c	[80,120]	[150,175]	0	570.31	300	216.58	13.96	32.15	7.62	76.29	6,20	0.7937	4.30
DP-MS	100u	Yes	1c	[80,120]	[150,175]	0	520.10	250	216.48	14.13	30.66	8.84	150.25	5,23	0.9455	3.74
DP-CG	100u	Yes	1c	[80,120]	[150,175]	0	559.01	300	206.47	14.00	29.19	9.36	179.27	6,24	0.7535	3.58
ST	100u	Yes	1c	[80,120]	[150,175]	1	1393.32	1000	301.89	18.79	64.04	8.60	236.49	20,20	0.3551	6.00
MT	100u	Yes	1c	[80,120]	[150,175]	1	766.18	400	289.98	19.56	42.94	13.70	369.26	8,32	0.7994	3.75
DP-MS	100u	Yes	1c	[80,120]	[150,175]	1	717.76	350	288.91	19.06	48.02	11.77	494.16	7,28	0.9275	4.29
DP-CG	100u	Yes	1c	[80,120]	[150,175]	1	757.45	400	281.06	18.98	44.41	13.01	672.02	8,31	0.7836	3.87
ST	100u	Yes	1c	[80,120]	[150,175]	2	1310.72	950	277.03	17.58	57.52	8.59	236.12	19,19	0.3418	6.11
MT	100u	Yes	1c	[80,120]	[150,175]	2	753.54	400	276.01	18.60	47.16	11.77	134.56	8,27	0.7805	4.30
DP-MS	100u	Yes	1c	[80,120]	[150,175]	2	713.08	350	286.07	19.98	42.70	14.33	456.18	7,35	0.9050	3.31
DP-CG	100u	Yes	1c	[80,120]	[150,175]	2	759.03	400	280.06	20.21	45.77	12.99	553.15	8,31	0.7897	3.74
ST	100u	Yes	1c	[80,120]	[150,175]	3	961.97	700	193.96	12.83	49.31	5.86	67.59	14,14	0.3419	6.00
MT	100u	Yes	1c	[80,120]	[150,175]	3	507.31	250	202.84	13.92	31.96	8.59	59.91	5,21	0.9055	4.00
DP-MS	100u	Yes	1c	[80,120]	[150,175]	3	502.71	250	195.84	14.56	33.79	8.52	153.75	5,23	0.8946	3.65
DP-CG	100u	Yes	1c	[80,120]	[150,175]	3	507.01	250	201.31	14.81	32.09	8.79	141.17	5,23	0.9056	3.65

ST	100u	Yes	1c	[80,120]	[150,175]	4	1167.10	850	250.39	14.94	44.91	6.87	212.23	17,17	0.3335	5.53
MT	100u	Yes	1c	[80,120]	[150,175]	4	671.49	350	261.26	15.92	35.88	8.45	125.21	7,22	0.8070	4.27
DP-MS	100u	Yes	1c	[80,120]	[150,175]	4	622.79	300	264.25	16.04	32.35	10.14	195.78	6,26	0.9357	3.62
DP-CG	100u	Yes	1c	[80,120]	[150,175]	4	619.25	300	260.83	15.81	32.63	9.99	195.06	6,25	0.9263	3.76
ST	100u	Yes	1c	[80,120]	[150,175]	5	1559.10	1150	326.86	18.98	54.18	9.09	221.01	23,23	0.3164	5.13
MT	100u	Yes	1c	[80,120]	[150,175]	5	783.37	400	307.90	20.10	42.89	12.48	198.94	8,32	0.8383	3.69
DP-MS	100u	Yes	1c	[80,120]	[150,175]	5	778.43	400	304.53	18.79	42.60	12.51	421.39	8,30	0.8263	3.93
DP-CG	100u	Yes	1c	[80,120]	[150,175]	5	774.83	400	299.40	19.25	43.99	12.19	527.13	8,31	0.8214	3.81
ST	100u	Yes	1c	[80,120]	[150,175]	6	1343.91	950	302.92	18.38	64.79	7.82	205.30	19,19	0.3752	6.11
MT	100u	Yes	1c	[80,120]	[150,175]	6	692.12	350	271.78	17.81	39.42	13.11	221.05	7,31	0.8515	3.74
DP-MS	100u	Yes	1c	[80,120]	[150,175]	6	682.17	350	260.56	17.75	42.29	11.57	446.06	7,27	0.8348	4.30
DP-CG	100u	Yes	1c	[80,120]	[150,175]	6	680.83	350	260.61	18.50	38.75	12.98	550.07	7,31	0.8249	3.74
ST	100u	Yes	1c	[80,120]	[150,175]	7	1299.51	950	271.60	17.38	52.29	8.24	234.12	19,19	0.3299	5.68
MT	100u	Yes	1c	[80,120]	[150,175]	7	684.96	350	262.20	18.96	42.92	10.88	153.21	7,27	0.8453	4.00
DP-MS	100u	Yes	1c	[80,120]	[150,175]	7	676.63	350	254.31	18.58	41.61	12.12	337.59	7,29	0.8205	3.72
DP-CG	100u	Yes	1c	[80,120]	[150,175]	7	684.34	350	262.08	18.59	42.14	11.53	381.21	7,28	0.8409	3.86
ST	100u	Yes	1c	[80,120]	[150,175]	8	1362.04	1000	286.91	17.04	49.60	8.49	230.34	20,20	0.3224	5.60
MT	100u	Yes	1c	[80,120]	[150,175]	8	704.52	350	281.23	18.00	45.44	9.86	245.95	7,24	0.8961	4.67
DP-MS	100u	Yes	1c	[80,120]	[150,175]	8	684.76	350	263.21	18.13	41.39	12.04	368.15	7,30	0.8392	3.73
DP-CG	100u	Yes	1c	[80,120]	[150,175]	8	699.25	350	276.88	18.15	43.31	10.91	469.18	7,27	0.8787	4.15
ST	100u	Yes	1c	[80,120]	[150,175]	9	1579.97	1200	310.75	17.18	42.09	9.95	200.05	24,24	0.2775	4.29
MT	100u	Yes	1c	[80,120]	[150,175]	9	711.00	350	295.38	18.51	32.67	14.43	171.73	7,37	0.8861	2.78
DP-MS	100u	Yes	1c	[80,120]	[150,175]	9	705.23	350	290.54	17.81	34.08	12.80	301.82	7,32	0.8771	3.22
DP-CG	100u	Yes	1c	[80,120]	[150,175]	9	700.29	350	283.99	18.02	36.46	11.82	332.38	7,30	0.8707	3.43
ST	100u	Yes	1c	[80,120]	[75,100]	0	693.12	450	170.14	11.88	56.88	4.23	30.97	9,9	0.5060	9.22
MT	100u	Yes	1c	[80,120]	[75,100]	0	376.16	200	124.38	12.08	31.56	8.14	49.89	4,18	0.7910	4.61
DP-MS	100u	Yes	1c	[80,120]	[75,100]	0	366.26	200	114.56	12.94	29.32	9.45	243.30	4,22	0.7415	3.77
DP-CG	100u	Yes	1c	[80,120]	[75,100]	0	370.98	200	118.99	12.92	29.64	9.44	280.30	4,22	0.7618	3.77
ST	100u	Yes	1c	[80,120]	[75,100]	1	844.27	550	188.08	16.25	85.00	4.95	52.37	11,11	0.5151	10.00
MT	100u	Yes	1c	[80,120]	[75,100]	1	458.94	250	139.78	15.08	43.13	10.95	239.57	5,24	0.7570	4.58
DP-MS	100u	Yes	1c	[80,120]	[75,100]	1	466.70	250	145.88	15.21	44.88	10.73	476.43	5,24	0.7867	4.58
DP-CG	100u	Yes	1c	[80,120]	[75,100]	1	465.06	250	144.60	14.90	44.74	10.83	502.48	5,24	0.7802	4.58
ST	100u	Yes	1c	[80,120]	[75,100]	2	826.58	500	197.63	17.50	106.88	4.57	138.73	10,10	0.6403	11.70
MT	100u	Yes	1c	[80,120]	[75,100]	2	448.31	250	127.58	17.55	37.41	15.77	481.57	5,35	0.7001	3.34
DP-MS	100u	Yes	1c	[80,120]	[75,100]	2	444.85	250	124.94	17.44	35.81	16.66	1018.11	5,37	0.6827	3.16
DP-CG	100u	Yes	1c	[80,120]	[75,100]	2	401.98	200	129.74	17.88	39.64	14.73	1038.08	4,33	0.9000	3.55
ST	100u	Yes	1c	[80,120]	[75,100]	3	593.28	400	123.48	10.63	55.31	3.87	33.18	8,8	0.4633	10.13
MT	100u	Yes	1c	[80,120]	[75,100]	3	280.92	150	83.30	11.43	27.10	9.10	98.43	3,20	0.7838	4.05
DP-MS	100u	Yes	1c	[80,120]	[75,100]	3	280.84	150	83.37	11.04	27.34	9.09	331.73	3,20	0.7830	4.05
DP-CG	100u	Yes	1c	[80,120]	[75,100]	3	280.89	150	83.25	11.06	27.22	9.36	343.66	3,21	0.7815	3.86
ST	100u	Yes	1c	[80,120]	[75,100]	4	765.41	500	180.78	13.33	66.67	4.63	225.66	10,10	0.5013	9.30
MT	100u	Yes	1c	[80,120]	[75,100]	4	443.51	250	130.92	13.40	40.27	8.93	181.51	5,20	0.7047	4.65
DP-MS	100u	Yes	1c	[80,120]	[75,100]	4	390.55	200	133.44	13.21	33.56	10.34	329.36	4,23	0.8483	4.04
DP-CG	100u	Yes	1c	[80,120]	[75,100]	4	392.05	200	133.22	14.35	33.69	10.79	358.38	4,25	0.8553	3.72
ST	100u	Yes	1c	[80,120]	[75,100]	5	823.95	550	158.60	15.50	94.50	5.35	56.69	11,11	0.4903	10.82
MT	100u	Yes	1c	[80,120]	[75,100]	5	445.03	250	122.10	16.46	45.10	11.37	520.86	5,25	0.7148	4.76
DP-MS	100u	Yes	1c	[80,120]	[75,100]	5	395.31	200	121.55	16.67	44.22	12.87	920.97	4,29	0.8870	4.10
DP-CG	100u	Yes	1c	[80,120]	[75,100]	5	399.99	200	127.10	16.46	43.85	12.58	913.45	4,28	0.9065	4.25
ST	100u	Yes	1c	[80,120]	[75,100]	6	736.08	500	151.14	13.33	66.88	4.73	70.89	10,10	0.4524	9.80
MT	100u	Yes	1c	[80,120]	[75,100]	6	382.72	200	117.69	13.50	43.13	8.41	113.54	4,18	0.8443	5.44
DP-MS	100u	Yes	1c	[80,120]	[75,100]	6	379.81	200	117.39	14.38	37.76	10.28	399.51	4,23	0.8150	4.26
DP-CG	100u	Yes	1c	[80,120]	[75,100]	6	378.16	200	114.84	14.17	39.69	9.47	381.30	4,21	0.8151	4.67
ST	100u	Yes	1c	[80,120]	[75,100]	7	837.28	550	190.19	15.00	76.88	5.21	238.42	11,11	0.4970	9.82
MT	100u	Yes	1c	[80,120]	[75,100]	7	468.06	250	147.99	15.21	44.69	10.18	201.64	5,23	0.7928	4.70
DP-MS	100u	Yes	1c	[80,120]	[75,100]	7	467.57	250	151.70	14.79	40.05	11.03	455.35	5,24	0.7799	4.50
DP-CG	100u	Yes	1c	[80,120]	[75,100]	7	467.25	250	149.11	15.63	41.88	10.64	814.95	5,24	0.7845	4.50
ST	100u	Yes	1c	[80,120]	[75,100]	8	686.05	450	160.69	12.08	58.96	4.32	34.99	9,9	0.4949	9.44
MT	100u	Yes	1c	[80,120]	[75,100]	8	379.92	200	123.91	13.00	34.13	8.88	68.02	4,20	0.8108	4.25
DP-MS	100u	Yes	1c	[80,120]	[75,100]	8	374.31	200	120.42	12.04	32.57	9.28	262.18	4,20	0.7806	4.25
DP-CG	100u	Yes	1c	[80,120]	[75,100]	8	370.97	200	114.59	12.19	35.99	8.20	282.69	4,18	0.7786	4.72
ST	100u	Yes	1c	[80,120]	[75,100]	9	659.92	450	138.67	10.92	56.00	4.33	26.45	9,9	0.4427	9.56
MT	100u	Yes	1c	[80,120]	[75,100]	9	354.92	200	103.17	11.43	31.05	9.28	103.41	4,21	0.6953	4.10
DP-MS	100u	Yes	1c	[80,120]	[75,100]	9	351.31	200	100.06	11.38	30.45	9.42	254.43	4,21	0.6783	4.10
DP-CG	100u	Yes	1c	[80,120]	[75,100]	9	350.72	200	99.78	11.06	30.24	9.64	284.07	4,21	0.6739	4.10
ST	100u	Yes	1c	[30,70]	[150,175]	0	847.13	650	159.45	9.36	23.47	4.84	4.40	13,13	0.2676	4.38
MT	100u	Yes	1c	[30,70]	[150,175]	0	415.63	200	179.74	11.00	19.17	5.72	16.72	4,18	0.9375	3.17
DP-MS	100u	Yes	1c	[30,70]	[150,175]	0	406.37	200	170.19	10.00	19.65	6.52	43.78	4,18	0.8945	3.17
DP-CG	100u	Yes	1c	[30,70]	[150,175]	0	412.45	200	175.42	10.75	19.74	6.55	47.25	4,19	0.9214	3.00
ST	100u	Yes	1c	[30,70]	[150,175]	1	482.52	350	107.52	6.46	16.04	2.50	0.67	7,7	0.3364	4.71
MT	100u	Yes	1c	[30,70]	[150,175]	1	270.89	150	98.84	6.04	12.60	3.40	2.88	3,9	0.7045	3.67
DP-MS	100u	Yes	1c	[30,70]	[150,175]	1	270.96	150	98.49	6.46	12.50	3.51	8.96	3,10	0.7052	3.30
DP-CG	100u	Yes	1c	[30,70]	[150,175]	1	272.52	150	100.47	6.15	12.19	3.72	15.70	3,10	0.7109	3.30
ST	100u	Yes	1c	[30,70]	[150,175]	2	857.95	650	168.03	9.52	25.15	5.24	12.82	13,13	0.2821	4.77
MT	100u	Yes	1c	[30,70]	[150,175]	2	471.42	250	180.61	10.89	24.00	5.91	17.69	5,16	0.7765	3.88
DP-MS	100u	Yes	1c	[30,70]	[150,175]	2	464.26	250	174.27	10.03	23.70	6.26	52.36	5,16	0.7496	3.88
DP-CG	100u	Yes	1c	[30,70]	[150,175]	2	462.58	250	172.43	10.49	23.78	5.87	144.08	5,16	0.7461	3.88
ST	100u	Yes	1c	[30,70]	[150,175]	3	635.23	450	154.60	8.38	19.39	2.87	0.66	9,9	0.3634	4.22
MT	100u	Yes	1c	[30,70]	[150,175]	3	382.51	200	153.25	10.42	15.00	3.84	5.76	4,14	0.7974	2.71
DP-MS	100u	Yes	1c	[30,70]	[150,175]	3	367.89	200	139.55	8.90	15.35	4.09	12.45	4,13	0.7330	2.92
DP-CG	100u	Yes	1c	[30,70]	[150,175]	3	368.51	200	141.28	8.58	14.57	4.08	20.84	4,12	0.7334	3.17
ST	100u	Yes	1c	[30,70]	[150,175]	4	702.55	500	164.70	9.75	24.42	3.69	2.26	10,10	0.3599	4.70
MT	100u	Yes	1c	[30,70]	[150,175]	4	390.72	200	155.72	10.50	19.56	4.94	6.25	4,14	0.8367	3.36
DP-MS	100u	Yes	1c	[30,70]	[150,175]	4	385.52	200	151.16	10.06	19.01	5.29	22.73	4,14	0.8115	3.36
DP-CG	100u	Yes	1c	[30,70]	[150,175]	4	385.12	200	151.44	9.44	18.86	5.38	22.55	4,14	0.8079	3.36
ST	100u	Yes	1c	[30,70]	[150,175]	5	739.29	550	152.70	9.25	22.63	4.72	1.69	11,11	0.3038	4.64
MT	100u	Yes	1c	[30,70]	[150,175]	5	396.90	200	160.08	9.88	21.81	5.14	11.95	4,13	0.8650	3.92
DP-MS	100u	Yes	1c	[30,70]	[150,175]	5	387.46	200	151.56	10.10	20.05	5.74	32.07	4,14	0.8200	3.64
DP-CG	100u	Yes	1c	[30,70]	[150,175]	5	387.85	200	151.44	10.28	20.35	5.79	32.98	4,15	0.8224	3.40

ST	100u	Yes	1c	[30,70]	[150,175]	6	568.63	450	94.60	5.69	14.31	4.02	1.46	9,9	0.2308	4.33
MT	100u	Yes	1c	[30,70]	[150,175]	6	278.11	150	104.71	6.25	12.77	4.38	6.79	3,11	0.7402	3.55
DP-MS	100u	Yes	1c	[30,70]	[150,175]	6	272.16	150	98.84	6.56	12.03	4.72	15.46	3,12	0.7041	3.25
DP-CG	100u	Yes	1c	[30,70]	[150,175]	6	271.16	150	96.58	7.19	13.23	4.17	18.35	3,11	0.7067	3.55
ST	100u	Yes	1c	[30,70]	[150,175]	7	405.18	300	84.69	4.83	13.25	2.40	0.57	6,6	0.3106	5.17
MT	100u	Yes	1c	[30,70]	[150,175]	7	250.25	150	80.99	5.33	10.88	3.05	2.39	3,8	0.5850	3.88
DP-MS	100u	Yes	1c	[30,70]	[150,175]	7	249.04	150	79.87	5.23	10.77	3.17	8.08	3,8	0.5770	3.88
DP-CG	100u	Yes	1c	[30,70]	[150,175]	7	208.06	100	89.97	5.00	10.00	3.09	9.36	2,8	0.9373	3.88
ST	100u	Yes	1c	[30,70]	[150,175]	8	611.46	450	131.17	7.92	19.06	3.31	1.64	9,9	0.3178	4.78
MT	100u	Yes	1c	[30,70]	[150,175]	8	369.07	200	138.92	8.96	17.50	3.69	12.78	4,11	0.7442	3.91
DP-MS	100u	Yes	1c	[30,70]	[150,175]	8	361.72	200	132.01	8.17	17.98	3.56	16.82	4,10	0.7135	4.30
DP-CG	100u	Yes	1c	[30,70]	[150,175]	8	364.06	200	133.77	8.44	18.49	3.36	17.69	4,10	0.7257	4.30
ST	100u	Yes	1c	[30,70]	[150,175]	9	676.91	500	142.01	8.73	22.26	3.91	2.29	10,10	0.3142	4.70
MT	100u	Yes	1c	[30,70]	[150,175]	9	385.16	200	150.36	10.00	20.42	4.38	7.35	4,12	0.8166	3.92
DP-MS	100u	Yes	1c	[30,70]	[150,175]	9	374.92	200	141.78	9.00	18.83	5.31	22.66	4,14	0.7647	3.36
DP-CG	100u	Yes	1c	[30,70]	[150,175]	9	372.71	200	139.41	9.10	18.78	5.42	29.69	4,14	0.7551	3.36
ST	100u	Yes	1c	[30,70]	[75,100]	0	279.20	200	55.45	4.38	17.50	1.87	0.79	4,4	0.3678	7.75
MT	100u	Yes	1c	[30,70]	[75,100]	0	174.57	100	53.56	5.00	13.44	2.57	2.58	2,6	0.6768	5.17
DP-MS	100u	Yes	1c	[30,70]	[75,100]	0	173.05	100	51.26	4.38	14.69	2.73	10.41	2,6	0.6654	5.17
DP-CG	100u	Yes	1c	[30,70]	[75,100]	0	173.86	100	52.05	4.38	14.69	2.75	10.28	2,6	0.6720	5.17
ST	100u	Yes	1c	[30,70]	[75,100]	1	378.82	250	82.33	7.08	37.08	2.33	4.79	5,5	0.4953	10.20
MT	100u	Yes	1c	[30,70]	[75,100]	1	196.15	100	64.09	6.88	20.94	4.25	23.00	2,9	0.8817	5.67
DP-MS	100u	Yes	1c	[30,70]	[75,100]	1	194.82	100	61.61	8.13	20.00	5.09	58.20	2,12	0.8649	4.25
DP-CG	100u	Yes	1c	[30,70]	[75,100]	1	190.51	100	58.73	7.40	19.22	5.16	62.32	2,12	0.8221	4.25
ST	100u	Yes	1c	[30,70]	[75,100]	2	353.34	250	72.23	5.42	23.33	2.36	4.00	5,5	0.3845	8.60
MT	100u	Yes	1c	[30,70]	[75,100]	2	236.47	150	59.41	5.98	17.72	3.35	14.18	3,8	0.5276	5.38
DP-MS	100u	Yes	1c	[30,70]	[75,100]	2	182.44	100	56.40	6.88	13.91	5.25	46.35	2,13	0.7298	3.31
DP-CG	100u	Yes	1c	[30,70]	[75,100]	2	182.15	100	55.10	7.19	15.09	4.76	47.62	2,13	0.7377	3.31
ST	100u	Yes	1c	[30,70]	[75,100]	3	460.10	300	112.45	8.13	36.88	2.66	10.54	6,6	0.4998	8.67
MT	100u	Yes	1c	[30,70]	[75,100]	3	269.31	150	84.52	7.83	22.79	4.16	15.86	3,10	0.7248	5.20
DP-MS	100u	Yes	1c	[30,70]	[75,100]	3	264.47	150	80.44	7.19	21.31	5.54	49.35	3,12	0.6844	4.33
DP-CG	100u	Yes	1c	[30,70]	[75,100]	3	265.99	150	83.40	8.23	18.91	5.45	86.77	3,13	0.6895	4.00
ST	100u	Yes	1c	[30,70]	[75,100]	4	355.92	250	71.75	5.83	26.04	2.30	3.18	5,5	0.3986	9.00
MT	100u	Yes	1c	[30,70]	[75,100]	4	192.69	100	63.96	6.00	18.75	3.99	3.78	2,9	0.8423	5.00
DP-MS	100u	Yes	1c	[30,70]	[75,100]	4	188.00	100	59.48	5.92	18.71	3.90	38.07	2,9	0.8035	5.00
DP-CG	100u	Yes	1c	[30,70]	[75,100]	4	186.86	100	59.33	6.39	16.32	4.82	45.91	2,11	0.7783	4.09
ST	100u	Yes	1c	[30,70]	[75,100]	5	540.18	350	126.84	10.00	50.00	3.34	15.62	7,7	0.5163	9.57
MT	100u	Yes	1c	[30,70]	[75,100]	5	340.54	200	95.12	10.31	27.50	7.61	40.36	4,17	0.6326	3.94
DP-MS	100u	Yes	1c	[30,70]	[75,100]	5	289.50	150	95.03	10.31	26.56	7.60	127.46	3,17	0.8352	3.94
DP-CG	100u	Yes	1c	[30,70]	[75,100]	5	288.14	150	93.54	10.00	27.66	6.94	124.59	3,15	0.8335	4.47
ST	100u	Yes	1c	[30,70]	[75,100]	6	493.13	350	96.81	7.71	35.42	3.20	13.62	7,7	0.3845	8.86
MT	100u	Yes	1c	[30,70]	[75,100]	6	266.20	150	79.22	7.63	24.31	5.05	38.29	3,11	0.7062	5.64
DP-MS	100u	Yes	1c	[30,70]	[75,100]	6	260.87	150	72.55	8.13	24.84	5.35	83.34	3,12	0.6778	5.17
DP-CG	100u	Yes	1c	[30,70]	[75,100]	6	259.53	150	71.72	7.92	24.53	5.36	85.74	3,12	0.6688	5.17
ST	100u	Yes	1c	[30,70]	[75,100]	7	261.84	200	43.03	3.54	13.33	1.93	0.99	4,4	0.2848	7.50
MT	100u	Yes	1c	[30,70]	[75,100]	7	164.73	100	44.26	4.00	14.25	2.22	3.52	2,5	0.5970	6.00
DP-MS	100u	Yes	1c	[30,70]	[75,100]	7	162.78	100	44.15	3.96	12.03	2.64	10.21	2,6	0.5678	5.00
DP-CG	100u	Yes	1c	[30,70]	[75,100]	7	161.05	100	42.67	3.96	11.93	2.50	10.53	2,6	0.5541	5.00
ST	100u	Yes	1c	[30,70]	[75,100]	8	384.89	250	86.38	7.50	38.75	2.26	5.23	5,5	0.5192	10.00
MT	100u	Yes	1c	[30,70]	[75,100]	8	244.27	150	61.26	7.81	20.00	5.20	43.81	3,12	0.5721	4.17
DP-MS	100u	Yes	1c	[30,70]	[75,100]	8	194.71	100	60.95	7.92	21.04	4.81	52.92	2,12	0.8699	4.17
DP-CG	100u	Yes	1c	[30,70]	[75,100]	8	196.06	100	60.95	7.50	23.44	4.17	67.17	2,10	0.8947	5.00
ST	100u	Yes	1c	[30,70]	[75,100]	9	487.54	350	98.78	7.08	28.33	3.34	8.90	7,7	0.3617	7.86
MT	100u	Yes	1c	[30,70]	[75,100]	9	278.24	150	93.99	8.13	20.73	5.39	16.85	3,13	0.7626	4.23
DP-MS	100u	Yes	1c	[30,70]	[75,100]	9	270.30	150	85.33	8.44	20.59	5.95	53.04	3,14	0.7160	3.93
DP-CG	100u	Yes	1c	[30,70]	[75,100]	9	267.47	150	84.32	7.22	20.02	5.92	75.04	3,13	0.6954	4.23
ST	50u	No	10c	[80,120]	[150,175]	0	869.21	480	261.87	21.99	68.96	36.39	10.04	16,16	0.4695	5.38
MT	50u	No	10c	[80,120]	[150,175]	0	498.85	180	186.49	21.00	23.77	87.59	92.46	6,38	0.8082	2.26
DP-MS	50u	No	10c	[80,120]	[150,175]	0	491.18	180	178.36	20.79	23.65	88.38	525.48	6,38	0.7797	2.26
DP-CG	50u	No	10c	[80,120]	[150,175]	0	492.50	180	179.27	21.29	25.04	86.90	443.12	6,39	0.7906	2.21
ST	50u	No	10c	[80,120]	[150,175]	1	1236.35	600	443.16	33.83	112.63	46.73	221.69	20,20	0.6262	6.00
MT	50u	No	10c	[80,120]	[150,175]	1	790.36	270	299.52	34.26	31.90	154.68	359.76	9,68	0.8494	1.76
DP-MS	50u	No	10c	[80,120]	[150,175]	1	771.78	270	291.97	34.20	34.48	141.13	1104.10	9,62	0.8396	1.94
DP-CG	50u	No	10c	[80,120]	[150,175]	1	766.69	270	291.80	33.33	36.58	134.97	954.85	9,58	0.8427	2.07
ST	50u	No	10c	[80,120]	[150,175]	2	1204.42	600	411.82	33.29	111.33	47.98	224.17	20,20	0.5926	5.80
MT	50u	No	10c	[80,120]	[150,175]	2	776.69	270	282.14	34.55	30.17	159.83	591.89	9,69	0.8067	1.68
DP-MS	50u	No	10c	[80,120]	[150,175]	2	751.99	270	280.71	34.38	37.18	129.72	1194.51	9,56	0.8226	2.07
DP-CG	50u	No	10c	[80,120]	[150,175]	2	754.71	270	283.83	34.00	36.30	130.58	1659.67	9,56	0.8260	2.07
ST	50u	No	10c	[80,120]	[150,175]	3	829.03	450	250.12	21.53	70.87	36.51	74.94	15,15	0.4875	5.60
MT	50u	No	10c	[80,120]	[150,175]	3	506.64	180	191.53	23.25	26.50	85.36	161.99	6,38	0.8457	2.21
DP-MS	50u	No	10c	[80,120]	[150,175]	3	488.73	180	171.35	22.97	25.07	89.34	465.61	6,39	0.7713	2.15
DP-CG	50u	No	10c	[80,120]	[150,175]	3	489.41	180	175.17	23.02	26.41	84.81	466.59	6,37	0.7898	2.27
ST	50u	No	10c	[80,120]	[150,175]	4	956.89	510	311.81	23.83	70.58	40.66	34.75	17,17	0.5057	5.53
MT	50u	No	10c	[80,120]	[150,175]	4	624.89	210	244.04	25.17	21.46	124.23	120.05	7,55	0.8638	1.71
DP-MS	50u	No	10c	[80,120]	[150,175]	4	601.02	210	233.22	24.98	28.88	103.94	532.53	7,46	0.8587	2.04
DP-CG	50u	No	10c	[80,120]	[150,175]	4	606.99	210	237.45	24.90	27.24	107.40	399.99	7,47	0.8647	2.00
ST	50u	No	10c	[80,120]	[150,175]	5	1249.94	690	380.20	30.44	94.29	55.02	229.06	23,23	0.4662	5.13
MT	50u	No	10c	[80,120]	[150,175]	5	743.97	240	279.02	31.90	24.94	168.10	310.02	8,74	0.8752	1.59
DP-MS	50u	No	10c	[80,120]	[150,175]	5	726.74	240	278.59	31.92	31.90	144.33	1257.05	8,64	0.8959	1.84
DP-CG	50u	No	10c	[80,120]	[150,175]	5	723.21	240	280.82	32.15	33.03	137.21	1049.49	8,61	0.9057	1.93
ST	50u	No	10c	[80,120]	[150,175]	6	1178.77	660	342.42	29.90	92.97	53.48	231.62	22,22	0.4509	5.27
MT	50u	No	10c	[80,120]	[150,175]	6	731.23	240	272.81	32.21	26.20	160.01	174.44	8,71	0.8646	1.63
DP-MS	50u	No	10c	[80,120]	[150,175]	6	704.61	240	273.48	31.35	35.45	124.33	1145.45	8,54	0.8925	2.15
DP-CG	50u	No	10c	[80,120]	[150,175]	6	702.06	240	263.33	31.41	33.17	134.15	1125.16	8,58	0.8601	2.00
ST	50u	No	10c	[80,120]	[150,175]	7	1117.08	570	371.58	30.96	99.71	44.83	212.82	19,19	0.5631	5.68
MT	50u	No	10c	[80,120]	[150,175]	7	700.27	240	258.86	31.04	28.33	142.03	267.55	8,61	0.8327	1.77
DP-MS	50u	No	10c	[80,120]	[150,175]	7	681.57	240	250.95	31.54	32.09	126.98	939.59	8,55	0.8262	1.96
DP-CG	50u	No	10c	[80,120]	[150,175]	7	678.41	240	258.35	30.69	37.01	112.35	710.71	8,48	0.8575	2.25

ST	50u	No	10c	[80,120]	[150,175]	8	1090.62	570	350.49	28.65	95.63	45.86	212.01	19,19	0.5324	5.89
MT	50u	No	10c	[80,120]	[150,175]	8	698.50	240	258.97	30.00	26.97	142.56	202.32	8,62	0.8255	1.81
DP-MS	50u	No	10c	[80,120]	[150,175]	8	675.12	240	251.77	29.85	34.90	118.60	1065.82	8,52	0.8318	2.15
DP-CG	50u	No	10c	[80,120]	[150,175]	8	677.36	240	257.66	29.85	36.72	113.13	1199.61	8,50	0.8522	2.24
ST	50u	No	10c	[80,120]	[150,175]	9	1256.53	720	373.40	28.42	77.25	57.45	29.30	24,24	0.4212	4.29
MT	50u	No	10c	[80,120]	[150,175]	9	744.17	240	297.42	32.61	22.85	151.29	207.62	8,72	0.9169	1.43
DP-MS	50u	No	10c	[80,120]	[150,175]	9	716.51	240	285.02	29.47	27.53	134.49	816.08	8,59	0.8907	1.75
DP-CG	50u	No	10c	[80,120]	[150,175]	9	717.51	240	281.83	30.61	26.28	138.79	829.06	8,62	0.8824	1.66
ST	50u	No	10c	[80,120]	[75,100]	0	662.27	270	244.39	22.50	103.75	21.63	30.44	9,9	0.8938	9.22
MT	50u	No	10c	[80,120]	[75,100]	0	439.75	150	143.88	23.46	23.35	99.06	121.42	5,43	0.8096	1.93
DP-MS	50u	No	10c	[80,120]	[75,100]	0	421.02	150	143.16	23.54	31.15	73.18	501.48	5,32	0.8461	2.59
DP-CG	50u	No	10c	[80,120]	[75,100]	0	422.24	150	140.18	23.02	30.78	78.25	361.01	5,34	0.8297	2.44
ST	50u	No	10c	[80,120]	[75,100]	1	795.58	360	244.67	28.33	133.33	29.25	41.81	12,12	0.7446	9.17
MT	50u	No	10c	[80,120]	[75,100]	1	527.04	210	154.50	28.96	44.94	88.64	253.35	7,37	0.7053	2.97
DP-MS	50u	No	10c	[80,120]	[75,100]	1	510.73	180	167.49	29.48	43.18	90.59	1135.49	6,39	0.8610	2.82
DP-CG	50u	No	10c	[80,120]	[75,100]	1	510.20	180	166.09	29.38	41.98	92.75	1074.22	6,40	0.8509	2.75
ST	50u	No	10c	[80,120]	[75,100]	2	838.23	390	238.39	32.50	146.25	31.09	75.21	13,13	0.7105	9.00
MT	50u	No	10c	[80,120]	[75,100]	2	537.00	210	150.28	32.88	50.92	92.93	475.98	7,39	0.7286	3.00
DP-MS	50u	No	10c	[80,120]	[75,100]	2	520.48	180	159.09	33.33	47.45	100.61	1250.15	6,43	0.8669	2.72
DP-CG	50u	No	10c	[80,120]	[75,100]	2	517.36	180	161.18	33.33	47.29	95.55	1058.41	6,41	0.8732	2.85
ST	50u	No	10c	[80,120]	[75,100]	3	565.11	270	156.29	21.25	96.25	21.32	15.10	9,9	0.6737	9.00
MT	50u	No	10c	[80,120]	[75,100]	3	347.68	120	105.69	22.50	30.42	69.07	128.78	4,31	0.8592	2.61
DP-MS	50u	No	10c	[80,120]	[75,100]	3	338.91	120	101.52	21.67	31.67	64.06	428.11	4,28	0.8409	2.89
DP-CG	50u	No	10c	[80,120]	[75,100]	3	338.15	120	99.94	21.96	31.33	64.92	434.55	4,29	0.8328	2.79
ST	50u	No	10c	[80,120]	[75,100]	4	727.21	360	214.29	24.58	99.38	28.96	45.73	12,12	0.6154	7.75
MT	50u	No	10c	[80,120]	[75,100]	4	463.26	180	143.16	25.04	37.02	78.04	262.37	6,33	0.7358	2.82
DP-MS	50u	No	10c	[80,120]	[75,100]	4	446.42	150	146.59	26.04	33.06	90.73	614.54	5,40	0.8818	2.33
DP-CG	50u	No	10c	[80,120]	[75,100]	4	448.41	150	151.78	26.04	32.81	87.78	711.53	5,39	0.9014	2.38
ST	50u	No	10c	[80,120]	[75,100]	5	821.19	390	229.49	30.63	139.38	31.70	126.11	13,13	0.6800	9.15
MT	50u	No	10c	[80,120]	[75,100]	5	492.80	150	142.07	33.50	33.38	133.85	621.66	5,60	0.9027	1.98
DP-MS	50u	No	10c	[80,120]	[75,100]	5	489.29	150	149.13	32.03	41.03	117.10	1418.19	5,51	0.9618	2.33
DP-CG	50u	No	10c	[80,120]	[75,100]	5	486.93	150	143.76	32.22	38.17	122.78	1396.21	5,54	0.9270	2.20
ST	50u	No	10c	[80,120]	[75,100]	6	717.87	330	216.87	25.83	118.33	26.84	27.96	11,11	0.7220	8.91
MT	50u	No	10c	[80,120]	[75,100]	6	453.59	150	133.05	28.75	27.31	114.48	175.42	5,52	0.8125	1.88
DP-MS	50u	No	10c	[80,120]	[75,100]	6	433.11	150	134.63	27.50	39.17	81.81	758.45	5,36	0.8719	2.72
DP-CG	50u	No	10c	[80,120]	[75,100]	6	431.45	150	134.61	27.60	42.08	77.16	881.54	5,34	0.8869	2.88
ST	50u	No	10c	[80,120]	[75,100]	7	802.89	360	255.50	28.75	130.00	28.64	60.62	12,12	0.7566	9.00
MT	50u	No	10c	[80,120]	[75,100]	7	531.24	210	160.44	29.08	41.92	89.80	431.79	7,38	0.7120	2.84
DP-MS	50u	No	10c	[80,120]	[75,100]	7	509.35	180	168.68	29.01	42.92	88.75	1015.02	6,38	0.8619	2.84
DP-CG	50u	No	10c	[80,120]	[75,100]	7	509.21	180	166.07	29.06	42.45	91.63	1138.46	6,39	0.8515	2.77
ST	50u	No	10c	[80,120]	[75,100]	8	656.75	300	208.66	23.33	100.42	24.34	19.71	10,10	0.7267	8.50
MT	50u	No	10c	[80,120]	[75,100]	8	431.96	150	138.99	25.00	27.29	90.67	217.68	5,40	0.8174	2.13
DP-MS	50u	No	10c	[80,120]	[75,100]	8	423.12	150	140.27	24.79	31.56	76.50	490.45	5,34	0.8428	2.50
DP-CG	50u	No	10c	[80,120]	[75,100]	8	422.43	150	138.89	23.99	32.45	77.10	581.32	5,33	0.8378	2.58
ST	50u	No	10c	[80,120]	[75,100]	9	624.84	270	209.70	21.25	101.88	22.02	30.27	9,9	0.8080	9.56
MT	50u	No	10c	[80,120]	[75,100]	9	386.09	120	123.56	22.50	23.00	97.03	196.22	4,42	0.9022	2.05
DP-MS	50u	No	10c	[80,120]	[75,100]	9	368.40	120	116.33	22.29	26.25	83.52	558.28	4,36	0.8851	2.39
DP-CG	50u	No	10c	[80,120]	[75,100]	9	377.26	120	123.39	22.71	26.66	84.51	530.36	4,37	0.9255	2.32
ST	50u	No	10c	[30,70]	[150,175]	0	689.28	420	184.21	14.44	37.12	33.50	6.84	14,14	0.3552	4.07
MT	50u	No	10c	[30,70]	[150,175]	0	372.71	120	136.68	15.54	11.07	89.43	38.08	4,41	0.8497	1.39
DP-MS	50u	No	10c	[30,70]	[150,175]	0	360.60	120	138.85	15.90	14.16	71.70	151.98	4,34	0.8821	1.68
DP-CG	50u	No	10c	[30,70]	[150,175]	0	363.92	120	137.40	15.56	13.35	77.60	132.48	4,36	0.8677	1.58
ST	50u	No	10c	[30,70]	[150,175]	1	368.50	210	108.63	8.63	24.60	16.65	0.52	7,7	0.4290	4.71
MT	50u	No	10c	[30,70]	[150,175]	1	224.40	90	78.45	9.67	9.67	36.62	3.96	3,17	0.6841	1.94
DP-MS	50u	No	10c	[30,70]	[150,175]	1	217.69	90	74.31	9.00	10.71	33.67	21.83	3,15	0.6597	2.20
DP-CG	50u	No	10c	[30,70]	[150,175]	1	220.25	90	76.71	9.04	10.73	33.77	21.52	3,15	0.6762	2.20
ST	50u	No	10c	[30,70]	[150,175]	2	692.63	390	216.93	15.83	39.69	30.18	6.87	13,13	0.4405	4.77
MT	50u	No	10c	[30,70]	[150,175]	2	418.66	150	163.76	16.00	15.50	73.40	36.91	5,32	0.8125	1.94
DP-MS	50u	No	10c	[30,70]	[150,175]	2	409.93	150	159.38	15.81	18.13	66.61	153.34	5,29	0.8072	2.14
DP-CG	50u	No	10c	[30,70]	[150,175]	2	407.82	150	157.10	15.50	17.13	68.10	155.48	5,29	0.7915	2.14
ST	50u	No	10c	[30,70]	[150,175]	3	503.94	270	169.12	11.65	32.46	20.72	0.79	9,9	0.4983	4.22
MT	50u	No	10c	[30,70]	[150,175]	3	272.73	90	106.56	12.33	8.96	54.88	8.18	3,26	0.8878	1.46
DP-MS	50u	No	10c	[30,70]	[150,175]	3	264.51	90	105.29	11.93	12.21	45.08	37.59	3,21	0.9031	1.81
DP-CG	50u	No	10c	[30,70]	[150,175]	3	267.42	90	104.53	11.88	10.50	50.51	34.88	3,23	0.8833	1.65
ST	50u	No	10c	[30,70]	[150,175]	4	602.38	330	192.41	14.33	39.19	26.45	1.53	11,11	0.4715	4.27
MT	50u	No	10c	[30,70]	[150,175]	4	355.81	120	135.90	15.17	11.25	73.50	15.01	4,32	0.8446	1.47
DP-MS	50u	No	10c	[30,70]	[150,175]	4	348.35	120	134.16	15.27	12.90	66.02	75.49	4,29	0.8469	1.62
DP-CG	50u	No	10c	[30,70]	[150,175]	4	349.91	120	134.86	14.77	12.84	67.44	77.21	4,29	0.8469	1.62
ST	50u	No	10c	[30,70]	[150,175]	5	641.92	360	200.52	15.63	36.98	28.80	1.34	12,12	0.4438	4.25
MT	50u	No	10c	[30,70]	[150,175]	5	374.92	120	146.39	17.17	12.13	79.24	21.01	4,36	0.9150	1.42
DP-MS	50u	No	10c	[30,70]	[150,175]	5	394.75	150	145.00	16.60	15.47	67.69	90.44	5,30	0.7403	1.70
DP-CG	50u	No	10c	[30,70]	[150,175]	5	394.00	150	143.17	16.39	15.40	69.04	98.92	5,30	0.7316	1.70
ST	50u	No	10c	[30,70]	[150,175]	6	465.34	270	135.51	10.38	28.38	21.08	0.89	9,9	0.4088	4.33
MT	50u	No	10c	[30,70]	[150,175]	6	264.73	90	101.59	11.25	9.13	52.77	5.49	3,24	0.8470	1.63
DP-MS	50u	No	10c	[30,70]	[150,175]	6	258.03	90	96.04	10.50	9.90	51.59	48.17	3,22	0.8102	1.77
DP-CG	50u	No	10c	[30,70]	[150,175]	6	259.55	90	96.42	10.79	9.33	53.01	58.55	3,23	0.8105	1.70
ST	50u	No	10c	[30,70]	[150,175]	7	332.71	180	111.75	7.50	19.38	14.08	0.48	6,6	0.4845	5.17
MT	50u	No	10c	[30,70]	[150,175]	7	214.47	90	72.42	8.00	7.44	36.61	6.03	3,17	0.6114	1.82
DP-MS	50u	No	10c	[30,70]	[150,175]	7	187.42	60	69.37	7.83	6.23	43.98	23.86	2,20	0.8695	1.55
DP-CG	50u	No	10c	[30,70]	[150,175]	7	190.52	60	70.77	8.13	5.94	45.69	25.01	2,22	0.8835	1.41
ST	50u	No	10c	[30,70]	[150,175]	8	482.31	270	151.15	11.15	28.75	21.27	1.15	9,9	0.4467	4.78
MT	50u	No	10c	[30,70]	[150,175]	8	280.12	90	102.73	12.36	8.29	66.74	18.35	3,32	0.8570	1.34
DP-MS	50u	No	10c	[30,70]	[150,175]	8	295.52	120	102.82	11.50	12.25	48.95	64.51	4,22	0.6625	1.95
DP-CG	50u	No	10c	[30,70]	[150,175]	8	277.51	90	105.72	11.67	10.13	60.00	58.55	3,27	0.8864	1.59
ST	50u	No	10c	[30,70]	[150,175]	9	552.56	300	176.61	14.25	38.71	23.00	1.03	10,10	0.4856	4.70
MT	50u	No	10c	[30,70]	[150,175]	9	332.71	120	125.69	15.13	13.25	58.65	25.13	4,27	0.8058	1.74
DP-MS	50u	No	10c	[30,70]	[150,175]	9	322.98	120	116.35	14.35	13.77	58.51	90.09	4,26	0.7575	1.81
DP-CG	50u	No	10c	[30,70]	[150,175]	9	325.77	120	117.82	14.85	14.36	58.73	103.14	4,27	0.7717	1.74

ST	50u	No	10c	[30,70]	[75,100]	0	262.02	120	89.37	8.33	34.58	9.73	0.78	4,4	0.7151	7.75
MT	50u	No	10c	[30,70]	[75,100]	0	163.16	60	52.70	9.33	10.42	30.71	11.88	2,14	0.7738	2.21
DP-MS	50u	No	10c	[30,70]	[75,100]	0	160.25	60	51.43	9.06	11.30	28.46	23.31	2,13	0.7688	2.38
DP-CG	50u	No	10c	[30,70]	[75,100]	0	160.23	60	48.06	9.09	9.91	33.17	26.47	2,15	0.7181	2.07
ST	50u	No	10c	[30,70]	[75,100]	1	375.46	180	109.34	13.75	58.13	14.25	4.57	6,6	0.6639	8.50
MT	50u	No	10c	[30,70]	[75,100]	1	235.62	90	71.79	13.96	22.50	37.37	42.42	3,16	0.7824	3.19
DP-MS	50u	No	10c	[30,70]	[75,100]	1	231.52	90	67.03	14.38	21.35	38.76	110.36	3,17	0.7446	3.00
DP-CG	50u	No	10c	[30,70]	[75,100]	1	232.36	90	66.65	14.17	20.42	41.13	121.27	3,18	0.7325	2.83
ST	50u	No	10c	[30,70]	[75,100]	2	333.62	150	113.44	11.25	47.50	11.43	1.57	5,5	0.7475	8.60
MT	50u	No	10c	[30,70]	[75,100]	2	193.41	60	63.49	11.63	12.06	46.23	26.17	2,21	0.9310	2.05
DP-MS	50u	No	10c	[30,70]	[75,100]	2	190.20	60	60.33	11.25	12.88	45.74	87.16	2,20	0.9049	2.15
DP-CG	50u	No	10c	[30,70]	[75,100]	2	189.29	60	58.47	11.04	11.90	47.88	77.88	2,21	0.8715	2.05
ST	50u	No	10c	[30,70]	[75,100]	3	425.86	180	152.35	15.00	65.00	13.51	3.64	6,6	0.8412	8.67
MT	50u	No	10c	[30,70]	[75,100]	3	273.83	90	90.42	15.88	14.13	63.41	31.44	3,30	0.8528	1.73
DP-MS	50u	No	10c	[30,70]	[75,100]	3	258.60	90	85.42	14.72	18.92	49.54	106.97	3,22	0.8498	2.36
DP-CG	50u	No	10c	[30,70]	[75,100]	3	257.54	90	83.12	14.93	18.52	50.96	88.65	3,23	0.8329	2.26
ST	50u	No	10c	[30,70]	[75,100]	4	359.05	180	106.98	11.13	46.56	14.38	1.56	6,6	0.5970	7.50
MT	50u	No	10c	[30,70]	[75,100]	4	199.83	60	63.12	12.00	11.25	53.45	24.48	2,24	0.9219	1.88
DP-MS	50u	No	10c	[30,70]	[75,100]	4	199.04	60	64.21	12.25	12.91	49.68	90.06	2,23	0.9565	1.96
DP-CG	50u	No	10c	[30,70]	[75,100]	4	198.87	60	65.49	11.67	12.79	48.92	92.75	2,22	0.9606	2.05
ST	50u	No	10c	[30,70]	[75,100]	5	539.92	240	177.44	20.00	83.75	18.73	7.91	8,8	0.7678	8.38
MT	50u	No	10c	[30,70]	[75,100]	5	350.70	120	116.14	20.50	23.25	70.81	82.17	4,31	0.8542	2.16
DP-MS	50u	No	10c	[30,70]	[75,100]	5	340.15	120	107.63	20.50	26.02	66.00	254.32	4,29	0.8289	2.31
DP-CG	50u	No	10c	[30,70]	[75,100]	5	340.56	120	108.06	20.25	25.13	67.13	289.68	4,29	0.8239	2.31
ST	50u	No	10c	[30,70]	[75,100]	6	439.05	210	131.48	15.00	66.25	16.32	5.86	7,7	0.6658	8.86
MT	50u	No	10c	[30,70]	[75,100]	6	258.76	90	76.62	15.17	19.00	57.98	58.32	3,25	0.7955	2.48
DP-MS	50u	No	10c	[30,70]	[75,100]	6	246.46	90	66.88	15.19	19.02	55.37	210.15	3,24	0.7309	2.58
DP-CG	50u	No	10c	[30,70]	[75,100]	6	248.30	90	72.23	15.21	21.93	48.93	269.97	3,21	0.7910	2.95
ST	50u	No	10c	[30,70]	[75,100]	7	238.97	120	75.36	7.08	26.88	9.65	0.47	4,4	0.5890	7.50
MT	50u	No	10c	[30,70]	[75,100]	7	151.77	60	43.47	7.92	8.13	32.26	6.63	2,15	0.6352	2.00
DP-MS	50u	No	10c	[30,70]	[75,100]	7	148.33	60	45.66	7.50	10.10	25.07	21.17	2,11	0.6766	2.73
DP-CG	50u	No	10c	[30,70]	[75,100]	7	148.62	60	44.44	7.85	9.55	26.79	30.44	2,13	0.6618	2.31
ST	50u	No	10c	[30,70]	[75,100]	8	373.21	180	106.99	13.33	58.33	14.56	1.97	6,6	0.6552	8.33
MT	50u	No	10c	[30,70]	[75,100]	8	235.28	90	69.08	14.33	20.96	40.91	49.80	3,18	0.7546	2.78
DP-MS	50u	No	10c	[30,70]	[75,100]	8	229.84	90	64.56	14.17	20.47	40.64	92.53	3,18	0.7191	2.78
DP-CG	50u	No	10c	[30,70]	[75,100]	8	230.91	90	63.65	14.38	20.00	42.88	120.98	3,19	0.7108	2.63
ST	50u	No	10c	[30,70]	[75,100]	9	459.56	210	162.90	13.75	55.63	17.28	5.28	7,7	0.7132	7.86
MT	50u	No	10c	[30,70]	[75,100]	9	281.80	90	98.49	15.00	14.94	63.37	48.10	3,28	0.9061	1.96
DP-MS	50u	No	10c	[30,70]	[75,100]	9	271.44	90	92.33	14.69	17.19	57.23	153.11	3,25	0.8812	2.20
DP-CG	50u	No	10c	[30,70]	[75,100]	9	274.85	90	91.63	14.78	16.83	61.61	152.42	3,27	0.8743	2.04
ST	50u	No	1c	[80,120]	[150,175]	0	828.97	510	235.05	20.17	59.69	4.06	151.67	17,17	0.3940	5.06
MT	50u	No	1c	[80,120]	[150,175]	0	364.84	150	165.22	21.88	13.02	14.71	120.42	5,67	0.8354	1.28
DP-MS	50u	No	1c	[80,120]	[150,175]	0	354.70	150	155.10	21.00	12.45	16.14	2221.05	5,70	0.7877	1.23
DP-CG	50u	No	1c	[80,120]	[150,175]	0	354.12	150	154.18	21.01	11.81	17.11	2444.82	5,75	0.7808	1.15
ST	50u	No	1c	[80,120]	[150,175]	1	1178.08	630	408.37	32.71	101.98	5.02	235.91	21,21	0.5493	5.71
MT	50u	No	1c	[80,120]	[150,175]	1	622.28	270	275.69	33.60	21.35	21.63	667.58	9,94	0.7653	1.28
DP-MS	50u	No	1c	[80,120]	[150,175]	1	582.75	240	265.42	33.17	19.35	24.81	5443.27	8,106	0.8277	1.13
DP-CG	50u	No	1c	[80,120]	[150,175]	1	583.55	240	266.12	33.19	19.53	24.73	4640.35	8,106	0.8300	1.13
ST	50u	No	1c	[80,120]	[150,175]	2	1171.61	660	373.63	33.33	99.38	5.27	231.05	22,22	0.4905	5.27
MT	50u	No	1c	[80,120]	[150,175]	2	571.12	240	253.90	35.03	20.75	21.46	507.10	8,95	0.8090	1.22
DP-MS	50u	No	1c	[80,120]	[150,175]	2	562.20	240	245.17	34.04	18.68	24.31	4893.19	8,104	0.7777	1.12
DP-CG	50u	No	1c	[80,120]	[150,175]	2	562.06	240	244.73	34.08	18.56	24.69	4635.70	8,106	0.7763	1.09
ST	50u	No	1c	[80,120]	[150,175]	3	800.23	450	252.69	21.75	72.16	3.62	91.70	15,15	0.4935	5.60
MT	50u	No	1c	[80,120]	[150,175]	3	367.88	150	165.39	22.83	16.85	12.81	225.92	5,56	0.8600	1.50
DP-MS	50u	No	1c	[80,120]	[150,175]	3	354.13	150	151.84	22.44	12.73	17.12	2147.07	5,74	0.7832	1.14
DP-CG	50u	No	1c	[80,120]	[150,175]	3	353.90	150	151.66	22.49	12.64	17.11	2401.26	5,74	0.7823	1.14
ST	50u	No	1c	[80,120]	[150,175]	4	912.27	510	304.63	23.63	69.92	4.10	126.30	17,17	0.4960	5.53
MT	50u	No	1c	[80,120]	[150,175]	4	482.49	210	215.22	24.96	14.97	17.33	257.17	7,76	0.7576	1.24
DP-MS	50u	No	1c	[80,120]	[150,175]	4	448.66	180	211.06	24.37	15.75	17.48	2095.29	6,76	0.8707	1.24
DP-CG	50u	No	1c	[80,120]	[150,175]	4	473.64	210	205.73	24.02	15.30	18.60	2344.55	7,79	0.7282	1.19
ST	50u	No	1c	[80,120]	[150,175]	5	1195.60	690	377.00	30.21	92.87	5.53	227.16	23,23	0.4616	5.13
MT	50u	No	1c	[80,120]	[150,175]	5	569.11	240	257.03	31.81	18.74	21.54	634.85	8,95	0.8005	1.24
DP-MS	50u	No	1c	[80,120]	[150,175]	5	536.40	210	253.37	31.08	19.22	22.73	4596.20	7,98	0.9036	1.20
DP-CG	50u	No	1c	[80,120]	[150,175]	5	560.67	240	247.39	31.09	18.66	23.52	3777.23	8,101	0.7740	1.17
ST	50u	No	1c	[80,120]	[150,175]	6	1136.16	660	348.31	30.09	92.44	5.32	228.82	22,22	0.4559	5.27
MT	50u	No	1c	[80,120]	[150,175]	6	557.42	240	245.21	31.98	18.63	21.61	573.67	8,96	0.7712	1.21
DP-MS	50u	No	1c	[80,120]	[150,175]	6	518.49	210	235.85	31.16	17.49	23.99	5691.60	7,103	0.8476	1.13
DP-CG	50u	No	1c	[80,120]	[150,175]	6	519.08	210	236.42	31.21	17.34	24.11	5773.01	7,104	0.8489	1.12
ST	50u	No	1c	[80,120]	[150,175]	7	1069.88	600	345.88	29.67	89.46	4.87	195.70	20,20	0.4948	5.40
MT	50u	No	1c	[80,120]	[150,175]	7	519.55	210	238.28	32.45	19.01	19.81	396.52	7,90	0.8646	1.20
DP-MS	50u	No	1c	[80,120]	[150,175]	7	504.94	210	224.11	30.85	17.95	22.02	3749.88	7,94	0.8146	1.15
DP-CG	50u	No	1c	[80,120]	[150,175]	7	505.20	210	224.18	30.89	18.27	21.85	3863.01	7,94	0.8161	1.15
ST	50u	No	1c	[80,120]	[150,175]	8	1090.26	600	355.71	28.88	100.85	4.82	222.35	20,20	0.5179	5.60
MT	50u	No	1c	[80,120]	[150,175]	8	506.33	210	227.20	30.31	18.06	20.77	577.88	7,92	0.8219	1.22
DP-MS	50u	No	1c	[80,120]	[150,175]	8	499.07	210	219.44	29.67	16.82	23.14	5110.20	7,100	0.7930	1.12
DP-CG	50u	No	1c	[80,120]	[150,175]	8	498.81	210	219.18	29.75	16.57	23.32	5853.73	7,101	0.7916	1.11
ST	50u	No	1c	[80,120]	[150,175]	9	1221.88	750	362.23	28.79	74.91	5.96	201.08	25,25	0.3935	4.12
MT	50u	No	1c	[80,120]	[150,175]	9	571.33	240	264.40	30.38	16.90	19.64	315.80	8,88	0.8088	1.17
DP-MS	50u	No	1c	[80,120]	[150,175]	9	558.67	240	251.48	29.22	16.48	21.49	2380.44	8,92	0.7715	1.12
DP-CG	50u	No	1c	[80,120]	[150,175]	9	558.64	240	251.72	29.32	16.11	21.49	2406.14	8,92	0.7713	1.12
ST	50u	No	1c	[80,120]	[75,100]	0	641.54	300	216.76	23.75	98.75	2.29	37.02	10,10	0.7398	8.30
MT	50u	No	1c	[80,120]	[75,100]	0	298.98	120	125.28	24.45	14.11	15.13	344.27	4,69	0.8675	1.20
DP-MS	50u	No	1c	[80,120]	[75,100]	0	291.88	120	118.65	23.38	13.45	16.40	3169.70	4,71	0.8234	1.17
DP-CG	50u	No	1c	[80,120]	[75,100]	0	292.10	120	118.54	23.50	12.91	17.15	2985.98	4,75	0.8202	1.11
ST	50u	No	1c	[80,120]	[75,100]	1	801.77	360	277.01	28.75	133.13	2.88	172.01	12,12	0.7989	9.17
MT	50u	No	1c	[80,120]	[75,100]	1	362.29	150	143.46	30.23	19.74	18.85	877.72	5,83	0.8237	1.33
DP-MS	50u	No	1c	[80,120]	[75,100]	1	352.74	150	134.25	29.33	17.73	21.43	9286.67	5,92	0.7723	1.20
DP-CG	50u	No	1c	[80,120]	[75,100]	1	352.77	150	134.33	29.14	18.17	21.13	6347.26	5,90	0.7739	1.22

ST	50u	No	1c	[80,120]	[75,100]	2	809.86	390	238.00	32.50	146.25	3.11	42.71	13,13	0.7099	9.00
MT	50u	No	1c	[80,120]	[75,100]	2	362.79	150	137.22	33.63	21.88	20.07	1463.65	5,87	0.8264	1.34
DP-MS	50u	No	1c	[80,120]	[75,100]	2	356.72	150	131.06	33.08	19.44	23.14	11080.20	5,99	0.7868	1.18
DP-CG	50u	No	1c	[80,120]	[75,100]	2	356.56	150	130.88	33.08	19.89	22.72	12240.40	5,97	0.7883	1.21
ST	50u	No	1c	[80,120]	[75,100]	3	549.67	270	160.72	20.50	96.25	2.20	14.03	9,9	0.6815	9.00
MT	50u	No	1c	[80,120]	[75,100]	3	229.30	90	88.98	22.25	15.25	12.83	325.79	3,57	0.9057	1.42
DP-MS	50u	No	1c	[80,120]	[75,100]	3	221.04	90	81.30	21.84	13.20	14.70	3987.43	3,65	0.8340	1.25
DP-CG	50u	No	1c	[80,120]	[75,100]	3	221.23	90	81.33	21.94	12.94	15.02	4022.07	3,67	0.8329	1.21
ST	50u	No	1c	[80,120]	[75,100]	4	715.43	330	250.93	25.00	106.88	2.62	71.55	11,11	0.7560	8.45
MT	50u	No	1c	[80,120]	[75,100]	4	313.04	120	134.02	26.64	16.14	16.24	664.66	4,73	0.9375	1.27
DP-MS	50u	No	1c	[80,120]	[75,100]	4	304.43	120	125.90	25.62	15.04	17.88	5648.25	4,78	0.8836	1.19
DP-CG	50u	No	1c	[80,120]	[75,100]	4	304.69	120	126.41	25.55	14.66	18.07	4443.21	4,78	0.8833	1.19
ST	50u	No	1c	[80,120]	[75,100]	5	797.59	390	230.11	31.25	143.13	3.11	242.15	13,13	0.6893	9.15
MT	50u	No	1c	[80,120]	[75,100]	5	345.73	150	121.85	32.77	20.66	20.45	1964.40	5,90	0.7546	1.32
DP-MS	50u	No	1c	[80,120]	[75,100]	5	311.82	120	117.97	32.39	19.35	22.12	14026.30	4,96	0.9132	1.24
DP-CG	50u	No	1c	[80,120]	[75,100]	5	311.55	120	117.61	32.43	19.48	22.04	11510.50	4,96	0.9125	1.24
ST	50u	No	1c	[80,120]	[75,100]	6	697.57	330	219.92	26.25	118.75	2.65	41.47	11,11	0.7294	8.91
MT	50u	No	1c	[80,120]	[75,100]	6	301.20	120	118.72	28.25	17.93	16.30	714.56	4,73	0.8822	1.34
DP-MS	50u	No	1c	[80,120]	[75,100]	6	292.73	120	111.11	26.85	16.38	18.39	7133.94	4,79	0.8258	1.24
DP-CG	50u	No	1c	[80,120]	[75,100]	6	292.56	120	110.71	26.84	16.64	18.37	6074.03	4,79	0.8253	1.24
ST	50u	No	1c	[80,120]	[75,100]	7	777.82	360	257.41	28.33	129.17	2.91	39.92	12,12	0.7571	9.00
MT	50u	No	1c	[80,120]	[75,100]	7	367.73	150	149.85	30.27	18.83	18.77	895.42	5,84	0.8449	1.29
DP-MS	50u	No	1c	[80,120]	[75,100]	7	360.32	150	142.03	29.43	18.27	20.58	6919.17	5,89	0.8066	1.21
DP-CG	50u	No	1c	[80,120]	[75,100]	7	360.76	150	142.86	29.68	16.80	21.43	5890.42	5,94	0.8038	1.15
ST	50u	No	1c	[80,120]	[75,100]	8	644.80	300	215.53	23.75	103.13	2.39	16.94	10,10	0.7483	8.50
MT	50u	No	1c	[80,120]	[75,100]	8	300.68	120	125.02	24.75	17.19	13.72	346.62	4,60	0.8872	1.42
DP-MS	50u	No	1c	[80,120]	[75,100]	8	293.44	120	117.67	24.57	15.35	15.85	2755.33	4,69	0.8379	1.23
DP-CG	50u	No	1c	[80,120]	[75,100]	8	293.20	120	117.91	24.40	15.69	15.21	3609.75	4,66	0.8401	1.29
ST	50u	No	1c	[80,120]	[75,100]	9	612.62	300	195.22	21.67	93.33	2.40	22.66	10,10	0.6779	8.60
MT	50u	No	1c	[80,120]	[75,100]	9	277.88	120	106.15	22.88	14.44	14.41	282.47	4,64	0.7640	1.34
DP-MS	50u	No	1c	[80,120]	[75,100]	9	272.93	120	101.09	22.15	13.74	15.95	4375.37	4,69	0.7298	1.25
DP-CG	50u	No	1c	[80,120]	[75,100]	9	272.75	120	101.04	22.02	13.51	16.19	3288.44	4,69	0.7272	1.25
ST	50u	No	1c	[30,70]	[150,175]	0	660.58	390	210.18	15.25	42.19	2.96	2.97	13,13	0.4338	4.38
MT	50u	No	1c	[30,70]	[150,175]	0	286.13	120	130.99	15.83	9.36	9.95	55.86	4,46	0.8124	1.24
DP-MS	50u	No	1c	[30,70]	[150,175]	0	277.37	120	122.57	15.05	8.83	10.92	537.55	4,48	0.7621	1.19
DP-CG	50u	No	1c	[30,70]	[150,175]	0	276.49	120	121.56	15.30	8.40	11.23	414.18	4,50	0.7559	1.14
ST	50u	No	1c	[30,70]	[150,175]	1	367.61	240	95.76	8.64	21.30	1.91	0.51	8,8	0.3330	4.13
MT	50u	No	1c	[30,70]	[150,175]	1	185.01	90	74.19	9.13	6.77	4.93	7.71	3,22	0.6270	1.50
DP-MS	50u	No	1c	[30,70]	[150,175]	1	179.49	90	68.59	9.07	5.55	6.28	81.40	3,28	0.5791	1.18
DP-CG	50u	No	1c	[30,70]	[150,175]	1	179.49	90	68.59	9.07	5.55	6.28	54.61	3,28	0.5791	1.18
ST	50u	No	1c	[30,70]	[150,175]	2	673.35	390	222.02	15.38	42.85	3.10	13.89	13,13	0.4535	4.77
MT	50u	No	1c	[30,70]	[150,175]	2	336.00	150	148.60	16.38	11.30	9.72	64.49	5,44	0.7328	1.41
DP-MS	50u	No	1c	[30,70]	[150,175]	2	327.15	150	140.03	15.75	9.81	11.56	574.95	5,50	0.6879	1.24
DP-CG	50u	No	1c	[30,70]	[150,175]	2	327.13	150	139.85	15.78	9.31	12.19	607.86	5,53	0.6848	1.17
ST	50u	No	1c	[30,70]	[150,175]	3	490.82	300	148.58	10.85	28.98	2.41	0.58	10,10	0.3967	3.80
MT	50u	No	1c	[30,70]	[150,175]	3	238.04	120	92.02	12.08	6.54	7.39	12.19	4,35	0.5765	1.09
DP-MS	50u	No	1c	[30,70]	[150,175]	3	201.33	90	85.54	11.39	5.90	8.49	120.93	3,37	0.7144	1.03
DP-CG	50u	No	1c	[30,70]	[150,175]	3	202.70	90	87.02	11.33	6.40	7.95	73.76	3,35	0.7279	1.09
ST	50u	No	1c	[30,70]	[150,175]	4	570.08	300	209.39	14.88	43.48	2.34	2.00	10,10	0.5647	4.70
MT	50u	No	1c	[30,70]	[150,175]	4	286.85	120	133.15	15.63	9.69	8.39	14.75	4,38	0.8239	1.24
DP-MS	50u	No	1c	[30,70]	[150,175]	4	277.69	120	124.26	15.28	8.22	9.93	166.76	4,44	0.7682	1.07
DP-CG	50u	No	1c	[30,70]	[150,175]	4	277.11	120	123.63	15.21	8.51	9.76	171.53	4,43	0.7664	1.09
ST	50u	No	1c	[30,70]	[150,175]	5	614.75	330	223.76	16.46	41.98	2.55	1.58	11,11	0.5397	4.64
MT	50u	No	1c	[30,70]	[150,175]	5	319.62	150	133.35	16.78	8.87	10.61	33.64	5,48	0.6617	1.06
DP-MS	50u	No	1c	[30,70]	[150,175]	5	285.93	120	129.71	16.42	8.86	10.95	320.37	4,48	0.8065	1.06
DP-CG	50u	No	1c	[30,70]	[150,175]	5	286.46	120	130.36	16.33	8.99	10.78	231.69	4,47	0.8100	1.09
ST	50u	No	1c	[30,70]	[150,175]	6	445.24	270	137.89	10.29	24.92	2.14	1.33	9,9	0.4042	4.33
MT	50u	No	1c	[30,70]	[150,175]	6	209.22	90	94.10	11.25	7.08	6.79	12.93	3,31	0.7801	1.26
DP-MS	50u	No	1c	[30,70]	[150,175]	6	202.31	90	87.57	10.71	6.44	7.59	129.58	3,33	0.7267	1.18
DP-CG	50u	No	1c	[30,70]	[150,175]	6	201.83	90	87.04	10.69	6.26	7.85	70.92	3,34	0.7215	1.15
ST	50u	No	1c	[30,70]	[150,175]	7	325.91	180	115.17	7.92	21.46	1.37	0.75	6,6	0.5063	5.17
MT	50u	No	1c	[30,70]	[150,175]	7	176.68	90	68.50	8.08	5.15	4.95	8.40	3,23	0.5669	1.35
DP-MS	50u	No	1c	[30,70]	[150,175]	7	146.43	60	68.00	7.77	5.34	5.32	128.78	2,24	0.8439	1.29
DP-CG	50u	No	1c	[30,70]	[150,175]	7	144.62	60	66.54	7.67	5.23	5.18	97.00	2,23	0.8266	1.35
ST	50u	No	1c	[30,70]	[150,175]	8	450.83	240	164.16	11.33	33.50	1.83	0.94	8,8	0.5505	5.38
MT	50u	No	1c	[30,70]	[150,175]	8	219.31	90	102.44	12.06	7.92	6.89	33.19	3,32	0.8494	1.34
DP-MS	50u	No	1c	[30,70]	[150,175]	8	209.34	90	92.77	11.44	6.73	8.40	181.75	3,37	0.7699	1.16
DP-CG	50u	No	1c	[30,70]	[150,175]	8	209.40	90	92.80	11.51	6.71	8.38	145.37	3,37	0.7705	1.16
ST	50u	No	1c	[30,70]	[150,175]	9	533.49	300	178.50	14.17	38.54	2.28	1.53	10,10	0.4888	4.70
MT	50u	No	1c	[30,70]	[150,175]	9	260.17	120	108.10	14.77	8.26	9.05	29.20	4,41	0.6844	1.15
DP-MS	50u	No	1c	[30,70]	[150,175]	9	256.17	120	104.01	14.26	8.49	9.41	236.16	4,41	0.6622	1.15
DP-CG	50u	No	1c	[30,70]	[150,175]	9	256.55	120	104.34	14.52	7.76	9.93	231.11	4,44	0.6610	1.07
ST	50u	No	1c	[30,70]	[75,100]	0	257.37	120	92.68	8.75	35.00	0.93	0.65	4,4	0.7369	7.75
MT	50u	No	1c	[30,70]	[75,100]	0	125.66	60	44.88	9.69	5.88	5.21	14.27	2,25	0.6433	1.24
DP-MS	50u	No	1c	[30,70]	[75,100]	0	120.03	60	39.92	8.94	5.03	6.14	129.56	2,27	0.5738	1.15
DP-CG	50u	No	1c	[30,70]	[75,100]	0	120.03	60	39.90	9.00	4.81	6.32	170.19	2,28	0.5716	1.11
ST	50u	No	1c	[30,70]	[75,100]	1	374.66	180	121.98	13.75	57.50	1.43	4.08	6,6	0.7035	8.50
MT	50u	No	1c	[30,70]	[75,100]	1	152.25	60	58.93	15.14	10.46	7.71	135.30	2,36	0.9094	1.42
DP-MS	50u	No	1c	[30,70]	[75,100]	1	148.57	60	55.97	14.44	9.16	9.01	911.30	2,40	0.8546	1.28
DP-CG	50u	No	1c	[30,70]	[75,100]	1	148.00	60	55.44	14.35	9.14	9.06	796.51	2,40	0.8481	1.28
ST	50u	No	1c	[30,70]	[75,100]	2	330.72	150	120.79	10.83	47.92	1.18	1.81	5,5	0.7769	8.60
MT	50u	No	1c	[30,70]	[75,100]	2	142.53	60	55.95	11.81	8.31	6.46	30.07	2,30	0.8111	1.43
DP-MS	50u	No	1c	[30,70]	[75,100]	2	134.81	60	48.77	11.16	6.54	8.34	463.64	2,37	0.7089	1.16
DP-CG	50u	No	1c	[30,70]	[75,100]	2	134.71	60	48.65	11.10	6.58	8.37	365.76	2,37	0.7075	1.16
ST	50u	No	1c	[30,70]	[75,100]	3	416.84	180	155.49	15.00	65.00	1.35	4.21	6,6	0.8516	8.67
MT	50u	No	1c	[30,70]	[75,100]	3	202.44	90	78.55	15.48	9.51	8.90	70.34	3,41	0.7319	1.27
DP-MS	50u	No	1c	[30,70]	[75,100]	3	197.22	90	73.64	14.82	8.97	9.79	631.11	3,43	0.6892	1.21
DP-CG	50u	No	1c	[30,70]	[75,100]	3	197.27	90	73.66	14.94	8.54	10.13	646.41	3,45	0.6867	1.16

ST	50u	No	1c	[30,70]	[75,100]	4	337.48	150	124.41	11.25	50.63	1.19	2.81	5,5	0.8070	9.00
MT	50u	No	1c	[30,70]	[75,100]	4	143.95	60	56.33	12.19	8.81	6.62	39.61	2,30	0.8258	1.50
DP-MS	50u	No	1c	[30,70]	[75,100]	4	141.60	60	54.08	11.67	8.31	7.55	521.25	2,33	0.7904	1.36
DP-CG	50u	No	1c	[30,70]	[75,100]	4	140.43	60	53.36	11.73	7.63	7.71	457.26	2,34	0.7756	1.32
ST	50u	No	1c	[30,70]	[75,100]	5	528.26	240	182.64	20.00	83.75	1.87	33.45	8,8	0.7808	8.38
MT	50u	No	1c	[30,70]	[75,100]	5	261.97	120	96.08	20.94	12.85	12.10	159.27	4,54	0.6916	1.24
DP-MS	50u	No	1c	[30,70]	[75,100]	5	229.70	90	94.05	20.10	12.64	12.92	1441.61	3,56	0.8998	1.20
DP-CG	50u	No	1c	[30,70]	[75,100]	5	230.77	90	95.02	20.42	12.81	12.51	1229.73	3,55	0.9104	1.22
ST	50u	No	1c	[30,70]	[75,100]	6	418.16	210	126.31	14.25	65.88	1.72	10.90	7,7	0.6471	8.86
MT	50u	No	1c	[30,70]	[75,100]	6	188.92	90	62.90	15.52	11.31	9.20	168.50	3,41	0.6429	1.51
DP-MS	50u	No	1c	[30,70]	[75,100]	6	184.34	90	58.04	15.13	9.51	11.66	2040.94	3,51	0.5922	1.22
DP-CG	50u	No	1c	[30,70]	[75,100]	6	184.57	90	58.67	15.13	9.41	11.35	1076.07	3,50	0.5957	1.24
ST	50u	No	1c	[30,70]	[75,100]	7	228.12	120	72.99	7.08	27.08	0.96	1.00	4,4	0.5785	7.50
MT	50u	No	1c	[30,70]	[75,100]	7	118.45	60	40.68	7.81	5.81	4.15	13.75	2,19	0.5771	1.58
DP-MS	50u	No	1c	[30,70]	[75,100]	7	113.90	60	36.41	7.44	4.30	5.75	124.76	2,26	0.5109	1.15
DP-CG	50u	No	1c	[30,70]	[75,100]	7	113.98	60	36.43	7.23	4.58	5.74	113.27	2,25	0.5119	1.20
ST	50u	No	1c	[30,70]	[75,100]	8	363.98	180	110.69	13.75	58.13	1.42	4.05	6,6	0.6684	8.33
MT	50u	No	1c	[30,70]	[75,100]	8	152.71	60	59.85	14.93	10.36	7.57	80.45	2,35	0.9146	1.43
DP-MS	50u	No	1c	[30,70]	[75,100]	8	147.08	60	54.83	14.54	8.90	8.81	907.88	2,40	0.8412	1.25
DP-CG	50u	No	1c	[30,70]	[75,100]	8	146.62	60	54.61	14.35	8.45	9.21	673.23	2,41	0.8311	1.22
ST	50u	No	1c	[30,70]	[75,100]	9	436.31	210	155.04	14.17	55.42	1.68	4.18	7,7	0.6915	7.86
MT	50u	No	1c	[30,70]	[75,100]	9	210.79	90	86.18	15.75	9.71	9.15	76.67	3,43	0.7867	1.28
DP-MS	50u	No	1c	[30,70]	[75,100]	9	204.12	90	80.01	14.83	9.02	10.26	767.55	3,45	0.7322	1.22
DP-CG	50u	No	1c	[30,70]	[75,100]	9	203.38	90	79.40	14.57	9.01	10.41	604.95	3,45	0.7258	1.22
ST	50u	Yes	10c	[80,120]	[150,175]	0	898.28	510	266.38	23.21	63.10	35.59	44.57	17,17	0.4403	5.06
MT	50u	Yes	10c	[80,120]	[150,175]	0	556.06	210	223.38	21.96	35.83	64.89	43.05	7,29	0.8446	2.97
DP-MS	50u	Yes	10c	[80,120]	[150,175]	0	566.08	210	227.68	22.75	32.56	73.09	168.71	7,34	0.8480	2.53
DP-CG	50u	Yes	10c	[80,120]	[150,175]	0	557.74	210	229.86	21.88	37.81	58.20	146.70	7,26	0.8699	3.31
ST	50u	Yes	10c	[80,120]	[150,175]	1	1295.27	660	446.87	33.83	103.85	50.72	235.08	22,22	0.5627	5.45
MT	50u	Yes	10c	[80,120]	[150,175]	1	870.77	330	343.28	35.20	45.77	116.51	370.79	11,52	0.8082	2.31
DP-MS	50u	Yes	10c	[80,120]	[150,175]	1	840.51	300	345.18	33.96	47.17	114.21	593.90	10,50	0.8932	2.40
DP-CG	50u	Yes	10c	[80,120]	[150,175]	1	839.13	300	340.20	36.15	45.84	116.93	740.67	10,54	0.8854	2.22
ST	50u	Yes	10c	[80,120]	[150,175]	2	1226.09	600	438.03	34.58	108.13	45.35	222.20	20,20	0.6164	5.80
MT	50u	Yes	10c	[80,120]	[150,175]	2	826.93	300	333.15	35.04	50.15	108.60	168.23	10,48	0.8793	2.42
DP-MS	50u	Yes	10c	[80,120]	[150,175]	2	828.79	300	336.31	35.13	48.88	108.48	515.25	10,48	0.8826	2.42
DP-CG	50u	Yes	10c	[80,120]	[150,175]	2	826.37	300	338.08	35.29	52.25	100.75	645.71	10,45	0.8950	2.58
ST	50u	Yes	10c	[80,120]	[150,175]	3	829.87	450	254.62	22.08	67.81	35.35	56.60	15,15	0.4893	5.60
MT	50u	Yes	10c	[80,120]	[150,175]	3	565.19	210	224.86	23.13	37.60	69.61	56.58	7,31	0.8593	2.71
DP-MS	50u	Yes	10c	[80,120]	[150,175]	3	555.70	210	219.06	23.04	37.88	65.72	193.34	7,29	0.8435	2.90
DP-CG	50u	Yes	10c	[80,120]	[150,175]	3	562.63	210	226.70	22.81	38.80	64.31	226.54	7,28	0.8678	3.00
ST	50u	Yes	10c	[80,120]	[150,175]	4	1010.27	540	329.49	24.88	74.71	41.20	181.11	18,18	0.5044	5.22
MT	50u	Yes	10c	[80,120]	[150,175]	4	701.00	270	291.64	25.79	39.04	74.52	90.09	9,33	0.8282	2.85
DP-MS	50u	Yes	10c	[80,120]	[150,175]	4	697.51	270	286.89	25.63	39.79	75.20	209.84	9,34	0.8192	2.76
DP-CG	50u	Yes	10c	[80,120]	[150,175]	4	693.88	270	285.13	24.35	39.57	74.82	459.38	9,32	0.8112	2.94
ST	50u	Yes	10c	[80,120]	[150,175]	5	1276.21	720	380.41	31.25	88.96	55.59	236.79	24,24	0.4422	4.92
MT	50u	Yes	10c	[80,120]	[150,175]	5	854.53	330	342.16	33.71	48.00	100.66	189.97	11,47	0.8078	2.51
DP-MS	50u	Yes	10c	[80,120]	[150,175]	5	833.80	300	343.95	33.75	47.30	108.80	509.41	10,50	0.8905	2.36
DP-CG	50u	Yes	10c	[80,120]	[150,175]	5	822.95	300	341.98	32.50	50.33	98.13	638.05	10,44	0.8911	2.68
ST	50u	Yes	10c	[80,120]	[150,175]	6	1219.65	630	410.29	31.54	98.75	49.07	215.71	21,21	0.5459	5.52
MT	50u	Yes	10c	[80,120]	[150,175]	6	771.62	300	292.25	30.96	46.71	101.71	161.23	10,43	0.7787	2.70
DP-MS	50u	Yes	10c	[80,120]	[150,175]	6	779.25	270	324.43	33.42	45.94	105.47	531.55	9,49	0.9414	2.37
DP-CG	50u	Yes	10c	[80,120]	[150,175]	6	777.98	270	323.69	32.50	44.10	107.68	753.84	9,48	0.9321	2.42
ST	50u	Yes	10c	[80,120]	[150,175]	7	1150.81	630	349.53	30.98	90.77	49.52	221.57	21,21	0.4778	5.14
MT	50u	Yes	10c	[80,120]	[150,175]	7	758.78	270	304.67	34.08	43.96	106.07	85.68	9,49	0.8938	2.20
DP-MS	50u	Yes	10c	[80,120]	[150,175]	7	747.86	270	306.60	32.29	48.33	90.63	427.17	9,40	0.9053	2.70
DP-CG	50u	Yes	10c	[80,120]	[150,175]	7	743.57	270	299.08	32.40	47.53	94.56	494.95	9,42	0.8866	2.57
ST	50u	Yes	10c	[80,120]	[150,175]	8	1214.66	630	411.13	30.98	97.02	45.53	226.57	21,21	0.5439	5.33
MT	50u	Yes	10c	[80,120]	[150,175]	8	782.17	300	317.77	30.06	48.51	85.83	230.71	10,38	0.8320	2.95
DP-MS	50u	Yes	10c	[80,120]	[150,175]	8	759.81	270	317.72	30.25	47.16	94.68	392.80	9,42	0.9211	2.67
DP-CG	50u	Yes	10c	[80,120]	[150,175]	8	751.15	270	313.05	30.33	48.13	89.65	469.63	9,40	0.9136	2.80
ST	50u	Yes	10c	[80,120]	[150,175]	9	1269.98	720	383.72	29.29	80.73	56.24	99.04	24,24	0.4344	4.29
MT	50u	Yes	10c	[80,120]	[150,175]	9	808.14	300	336.90	30.71	37.56	102.97	167.16	10,46	0.8445	2.24
DP-MS	50u	Yes	10c	[80,120]	[150,175]	9	806.65	300	336.85	29.90	39.34	100.57	352.63	10,44	0.8468	2.34
DP-CG	50u	Yes	10c	[80,120]	[150,175]	9	807.97	300	340.15	30.42	39.92	97.49	445.18	10,43	0.8561	2.40
ST	50u	Yes	10c	[80,120]	[75,100]	0	674.02	300	229.64	23.00	97.75	23.63	29.01	10,10	0.7611	8.30
MT	50u	Yes	10c	[80,120]	[75,100]	0	463.30	180	160.13	23.96	40.52	58.69	61.21	6,26	0.8024	3.19
DP-MS	50u	Yes	10c	[80,120]	[75,100]	0	443.00	150	159.10	26.67	36.04	71.20	248.91	5,35	0.9499	2.37
DP-CG	50u	Yes	10c	[80,120]	[75,100]	0	459.77	180	153.41	24.38	38.02	63.96	219.84	6,29	0.7714	2.86
ST	50u	Yes	10c	[80,120]	[75,100]	1	844.43	390	270.62	28.75	123.75	31.31	194.39	13,13	0.7096	8.46
MT	50u	Yes	10c	[80,120]	[75,100]	1	558.88	210	190.80	30.00	56.56	71.52	171.63	7,31	0.8543	3.55
DP-MS	50u	Yes	10c	[80,120]	[75,100]	1	561.14	210	189.08	30.21	50.10	81.74	569.20	7,36	0.8271	3.06
DP-CG	50u	Yes	10c	[80,120]	[75,100]	1	558.19	210	189.11	29.58	53.65	75.86	576.20	7,33	0.8375	3.33
ST	50u	Yes	10c	[80,120]	[75,100]	2	845.21	390	245.37	32.50	146.25	31.09	178.51	13,13	0.7212	9.00
MT	50u	Yes	10c	[80,120]	[75,100]	2	565.89	210	182.22	33.00	55.25	85.42	345.82	7,36	0.8358	3.25
DP-MS	50u	Yes	10c	[80,120]	[75,100]	2	549.13	210	163.53	32.88	55.10	87.62	820.69	7,37	0.7814	3.16
DP-CG	50u	Yes	10c	[80,120]	[75,100]	2	553.27	210	171.01	32.92	57.29	82.05	962.75	7,35	0.8108	3.34
ST	50u	Yes	10c	[80,120]	[75,100]	3	577.79	270	171.64	20.63	93.75	21.78	11.58	9,9	0.6991	9.00
MT	50u	Yes	10c	[80,120]	[75,100]	3	357.24	120	113.59	22.50	31.85	69.30	131.41	4,31	0.9076	2.61
DP-MS	50u	Yes	10c	[80,120]	[75,100]	3	360.09	120	114.03	21.94	31.41	72.72	275.98	4,32	0.9035	2.53
DP-CG	50u	Yes	10c	[80,120]	[75,100]	3	372.07	150	103.66	21.50	33.79	63.12	243.10	5,27	0.6911	3.00
ST	50u	Yes	10c	[80,120]	[75,100]	4	751.12	360	238.07	25.00	99.38	28.67	137.57	12,12	0.6559	7.75
MT	50u	Yes	10c	[80,120]	[75,100]	4	493.30	180	168.21	26.17	38.10	80.82	92.34	6,36	0.8285	2.58
DP-MS	50u	Yes	10c	[80,120]	[75,100]	4	490.40	180	171.90	26.35	42.97	69.18	380.99	6,31	0.8619	3.00
DP-CG	50u	Yes	10c	[80,120]	[75,100]	4	486.41	180	167.75	25.73	46.51	66.42	407.51	6,29	0.8602	3.21
ST	50u	Yes	10c	[80,120]	[75,100]	5	848.63	420	231.71	30.42	132.08	34.42	81.79	14,14	0.6212	8.50
MT	50u	Yes	10c	[80,120]	[75,100]	5	520.99	180	160.49	33.71	53.15	93.65	263.85	6,42	0.8968	2.83
DP-MS	50u	Yes	10c	[80,120]	[75,100]	5	522.82	180	162.98	32.81	55.31	91.71	822.73	6,41	0.9105	2.90
DP-CG	50u	Yes	10c	[80,120]	[75,100]	5	519.32	180	165.28	31.88	51.67	90.49	732.48	6,39	0.8990	3.05

ST	50u	Yes	10c	[80,120]	[75,100]	6	725.31	330	225.10	25.83	117.50	26.88	107.54	11,11	0.7350	8.91
MT	50u	Yes	10c	[80,120]	[75,100]	6	480.80	180	156.92	27.08	47.60	69.20	163.34	6,30	0.8343	3.27
DP-MS	50u	Yes	10c	[80,120]	[75,100]	6	476.57	180	151.33	27.29	46.98	70.97	417.40	6,31	0.8139	3.16
DP-CG	50u	Yes	10c	[80,120]	[75,100]	6	474.82	180	148.63	28.13	47.92	70.15	420.85	6,32	0.8123	3.06
ST	50u	Yes	10c	[80,120]	[75,100]	7	836.73	390	262.75	29.17	124.17	30.65	202.01	13,13	0.6991	8.31
MT	50u	Yes	10c	[80,120]	[75,100]	7	558.27	210	195.08	29.17	53.33	70.69	120.63	7,30	0.8520	3.60
DP-MS	50u	Yes	10c	[80,120]	[75,100]	7	563.27	210	192.86	30.08	50.90	79.43	493.79	7,35	0.8402	3.09
DP-CG	50u	Yes	10c	[80,120]	[75,100]	7	555.53	210	188.59	28.96	51.88	76.11	512.75	7,32	0.8275	3.38
ST	50u	Yes	10c	[80,120]	[75,100]	8	684.67	300	234.50	23.75	102.50	23.92	23.95	10,10	0.7846	8.50
MT	50u	Yes	10c	[80,120]	[75,100]	8	465.53	180	155.02	24.58	36.88	69.05	86.49	6,30	0.7728	2.83
DP-MS	50u	Yes	10c	[80,120]	[75,100]	8	461.06	180	150.87	24.58	39.38	66.23	257.74	6,29	0.7694	2.93
DP-CG	50u	Yes	10c	[80,120]	[75,100]	8	466.51	180	158.70	24.38	40.83	62.60	260.82	6,27	0.8007	3.15
ST	50u	Yes	10c	[80,120]	[75,100]	9	627.15	270	210.84	21.25	103.13	21.93	20.36	9,9	0.8140	9.56
MT	50u	Yes	10c	[80,120]	[75,100]	9	416.65	150	143.06	22.25	35.92	65.43	63.24	5,28	0.8631	3.07
DP-MS	50u	Yes	10c	[80,120]	[75,100]	9	416.96	150	146.17	22.08	38.54	60.17	257.36	5,26	0.8878	3.31
DP-CG	50u	Yes	10c	[80,120]	[75,100]	9	408.42	150	133.78	22.13	37.40	65.12	286.53	5,28	0.8327	3.07
ST	50u	Yes	10c	[30,70]	[150,175]	0	699.83	390	224.89	16.07	41.48	27.39	2.39	13,13	0.4567	4.38
MT	50u	Yes	10c	[30,70]	[150,175]	0	442.60	180	172.97	16.85	22.18	50.60	22.10	6,25	0.7392	2.28
DP-MS	50u	Yes	10c	[30,70]	[150,175]	0	425.00	150	186.93	18.13	22.92	47.02	48.94	5,24	0.9529	2.38
DP-CG	50u	Yes	10c	[30,70]	[150,175]	0	442.51	180	176.78	16.15	23.18	46.41	103.06	6,22	0.7531	2.59
ST	50u	Yes	10c	[30,70]	[150,175]	1	377.59	210	118.99	8.54	23.33	16.72	0.57	7,7	0.4538	4.71
MT	50u	Yes	10c	[30,70]	[150,175]	1	280.64	120	110.90	9.38	16.25	24.12	3.12	4,11	0.7146	3.00
DP-MS	50u	Yes	10c	[30,70]	[150,175]	1	275.89	120	105.54	10.07	15.87	24.41	9.05	4,12	0.6898	2.75
DP-CG	50u	Yes	10c	[30,70]	[150,175]	1	272.96	120	104.10	9.58	15.42	23.86	13.16	4,11	0.6767	3.00
ST	50u	Yes	10c	[30,70]	[150,175]	2	693.02	390	216.64	15.29	40.02	31.07	8.27	13,13	0.4397	4.77
MT	50u	Yes	10c	[30,70]	[150,175]	2	467.45	180	193.12	16.39	24.64	53.30	21.29	6,24	0.8147	2.58
DP-MS	50u	Yes	10c	[30,70]	[150,175]	2	464.51	180	192.05	15.63	25.31	51.52	62.45	6,22	0.8107	2.82
DP-CG	50u	Yes	10c	[30,70]	[150,175]	2	460.39	180	184.10	16.37	23.16	56.76	82.75	6,25	0.7784	2.48
ST	50u	Yes	10c	[30,70]	[150,175]	3	534.56	300	175.50	11.77	25.10	22.19	1.00	10,10	0.4432	3.80
MT	50u	Yes	10c	[30,70]	[150,175]	3	359.00	150	145.65	13.13	15.73	34.49	3.47	5,17	0.7269	2.24
DP-MS	50u	Yes	10c	[30,70]	[150,175]	3	355.82	150	143.58	13.23	16.77	32.24	13.43	5,16	0.7243	2.38
DP-CG	50u	Yes	10c	[30,70]	[150,175]	3	328.50	120	145.39	13.96	17.50	31.65	15.13	4,17	0.9236	2.24
ST	50u	Yes	10c	[30,70]	[150,175]	4	615.46	330	205.70	15.38	40.06	24.32	1.43	11,11	0.5000	4.27
MT	50u	Yes	10c	[30,70]	[150,175]	4	432.57	180	169.33	16.38	19.92	46.95	9.76	6,22	0.7157	2.14
DP-MS	50u	Yes	10c	[30,70]	[150,175]	4	407.54	150	176.15	16.54	20.85	43.99	22.31	5,21	0.8916	2.24
DP-CG	50u	Yes	10c	[30,70]	[150,175]	4	416.06	150	184.51	17.25	21.88	42.43	34.20	5,21	0.9337	2.24
ST	50u	Yes	10c	[30,70]	[150,175]	5	653.86	360	212.18	16.04	37.60	28.04	1.94	12,12	0.4654	4.25
MT	50u	Yes	10c	[30,70]	[150,175]	5	438.13	180	171.75	16.67	22.50	47.21	8.32	6,21	0.7357	2.43
DP-MS	50u	Yes	10c	[30,70]	[150,175]	5	438.20	180	171.76	16.67	22.60	47.17	36.07	6,21	0.7362	2.43
DP-CG	50u	Yes	10c	[30,70]	[150,175]	5	418.56	150	180.37	17.29	21.77	49.13	44.99	5,23	0.9168	2.22
ST	50u	Yes	10c	[30,70]	[150,175]	6	470.26	270	143.89	10.42	24.79	21.16	1.11	9,9	0.4176	4.33
MT	50u	Yes	10c	[30,70]	[150,175]	6	298.23	120	116.51	11.13	14.63	35.97	5.25	4,16	0.7435	2.44
DP-MS	50u	Yes	10c	[30,70]	[150,175]	6	295.32	120	112.77	10.58	14.40	37.57	17.67	4,16	0.7200	2.44
DP-CG	50u	Yes	10c	[30,70]	[150,175]	6	296.29	120	110.99	10.79	13.58	40.92	17.25	4,18	0.7073	2.17
ST	50u	Yes	10c	[30,70]	[150,175]	7	339.33	180	116.22	7.92	21.46	13.73	0.54	6,6	0.5098	5.17
MT	50u	Yes	10c	[30,70]	[150,175]	7	226.31	90	90.70	7.50	12.50	25.61	2.70	3,11	0.7713	2.82
DP-MS	50u	Yes	10c	[30,70]	[150,175]	7	223.91	90	87.16	7.81	12.66	26.28	9.04	3,12	0.7517	2.58
DP-CG	50u	Yes	10c	[30,70]	[150,175]	7	226.51	90	92.65	7.57	13.16	23.13	8.80	3,10	0.7904	3.10
ST	50u	Yes	10c	[30,70]	[150,175]	8	478.91	240	173.04	12.08	36.25	17.53	1.80	8,8	0.5836	5.38
MT	50u	Yes	10c	[30,70]	[150,175]	8	325.49	120	142.99	12.29	20.63	29.59	4.90	4,14	0.9207	3.07
DP-MS	50u	Yes	10c	[30,70]	[150,175]	8	313.22	120	129.93	11.56	18.49	33.24	17.87	4,15	0.8375	2.87
DP-CG	50u	Yes	10c	[30,70]	[150,175]	8	318.04	120	134.06	12.29	20.31	31.37	15.60	4,15	0.8741	2.87
ST	50u	Yes	10c	[30,70]	[150,175]	9	562.90	300	184.68	14.79	41.46	21.97	2.10	10,10	0.5100	4.70
MT	50u	Yes	10c	[30,70]	[150,175]	9	390.09	150	160.00	15.50	21.33	43.26	7.58	5,20	0.8242	2.35
DP-MS	50u	Yes	10c	[30,70]	[150,175]	9	381.50	150	152.19	15.58	21.04	42.69	27.30	5,20	0.7919	2.35
DP-CG	50u	Yes	10c	[30,70]	[150,175]	9	386.75	150	160.50	15.63	22.50	38.12	28.24	5,18	0.8326	2.61
ST	50u	Yes	10c	[30,70]	[75,100]	0	266.55	120	93.45	8.75	35.00	9.35	0.74	4,4	0.7407	7.75
MT	50u	Yes	10c	[30,70]	[75,100]	0	169.88	60	61.64	10.00	14.06	24.18	3.55	2,12	0.9171	2.58
DP-MS	50u	Yes	10c	[30,70]	[75,100]	0	165.86	60	59.72	9.17	15.42	21.55	11.43	2,10	0.9045	3.10
DP-CG	50u	Yes	10c	[30,70]	[75,100]	0	169.84	60	61.63	10.00	14.38	23.84	13.87	2,12	0.9210	2.58
ST	50u	Yes	10c	[30,70]	[75,100]	1	384.85	180	118.16	13.33	58.75	14.61	2.75	6,6	0.6942	8.50
MT	50u	Yes	10c	[30,70]	[75,100]	1	247.69	90	81.24	13.96	20.94	41.55	17.69	3,18	0.8324	2.83
DP-MS	50u	Yes	10c	[30,70]	[75,100]	1	234.55	90	69.54	14.38	22.81	37.82	58.51	3,17	0.7735	3.00
DP-CG	50u	Yes	10c	[30,70]	[75,100]	1	238.79	90	73.22	13.75	21.56	40.26	65.00	3,17	0.7824	3.00
ST	50u	Yes	10c	[30,70]	[75,100]	2	350.19	150	127.49	11.25	50.00	11.45	2.57	5,5	0.8162	8.60
MT	50u	Yes	10c	[30,70]	[75,100]	2	221.89	90	71.29	11.88	20.00	28.73	11.87	3,13	0.7409	3.31
DP-MS	50u	Yes	10c	[30,70]	[75,100]	2	218.25	90	66.55	11.67	18.54	31.50	33.23	3,15	0.6954	2.87
DP-CG	50u	Yes	10c	[30,70]	[75,100]	2	218.86	90	67.43	11.25	18.75	31.43	32.19	3,14	0.6996	3.07
ST	50u	Yes	10c	[30,70]	[75,100]	3	426.89	180	157.15	13.75	61.25	14.74	3.87	6,6	0.8363	8.67
MT	50u	Yes	10c	[30,70]	[75,100]	3	300.51	120	101.70	14.75	23.81	40.25	22.27	4,18	0.7495	2.89
DP-MS	50u	Yes	10c	[30,70]	[75,100]	3	294.57	120	97.02	14.90	24.32	38.33	55.27	4,17	0.7302	3.06
DP-CG	50u	Yes	10c	[30,70]	[75,100]	3	296.24	120	97.79	14.58	22.81	41.05	65.51	4,18	0.7227	2.89
ST	50u	Yes	10c	[30,70]	[75,100]	4	359.64	180	110.89	10.83	43.13	14.79	5.85	6,6	0.5945	7.50
MT	50u	Yes	10c	[30,70]	[75,100]	4	224.90	90	71.50	11.88	20.21	31.32	8.80	3,14	0.7440	3.21
DP-MS	50u	Yes	10c	[30,70]	[75,100]	4	225.50	90	71.47	11.56	18.18	34.29	37.18	3,15	0.7243	3.00
DP-CG	50u	Yes	10c	[30,70]	[75,100]	4	226.62	90	71.45	11.88	20.00	33.30	42.92	3,15	0.7419	3.00
ST	50u	Yes	10c	[30,70]	[75,100]	5	559.06	240	196.58	20.00	83.75	18.73	4.59	8,8	0.8157	8.38
MT	50u	Yes	10c	[30,70]	[75,100]	5	383.19	150	126.68	21.25	32.63	52.64	32.45	5,24	0.7761	2.79
DP-MS	50u	Yes	10c	[30,70]	[75,100]	5	377.04	150	124.06	20.00	34.27	48.71	126.56	5,21	0.7676	3.19
DP-CG	50u	Yes	10c	[30,70]	[75,100]	5	377.52	150	120.48	20.63	31.98	54.44	168.37	5,24	0.7449	2.79
ST	50u	Yes	10c	[30,70]	[75,100]	6	448.45	210	139.00	15.00	68.13	16.32	14.42	7,7	0.6940	8.86
MT	50u	Yes	10c	[30,70]	[75,100]	6	295.47	120	91.58	16.04	30.21	37.64	37.53	4,17	0.7470	3.65
DP-MS	50u	Yes	10c	[30,70]	[75,100]	6	293.81	120	86.97	16.21	27.56	43.07	100.37	4,20	0.7084	3.10
DP-CG	50u	Yes	10c	[30,70]	[75,100]	6	293.42	120	87.32	15.38	29.19	41.54	111.72	4,18	0.7151	3.44
ST	50u	Yes	10c	[30,70]	[75,100]	7	243.69	120	79.39	7.50	27.50	9.30	0.57	4,4	0.6157	7.50
MT	50u	Yes	10c	[30,70]	[75,100]	7	156.62	60	55.92	7.92	15.42	17.36	3.14	2,8	0.8509	3.75
DP-MS	50u	Yes	10c	[30,70]	[75,100]	7	151.75	60	49.50	8.65	12.81	20.79	11.49	2,11	0.7633	2.73
DP-CG	50u	Yes	10c	[30,70]	[75,100]	7	152.73	60	50.18	7.92	11.56	23.07	12.82	2,11	0.7453	2.73

ST	50u	Yes	10c	[30,70]	[75,100]	8	380.64	180	113.97	13.75	58.75	14.17	9.48	6,6	0.6820	8.33
MT	50u	Yes	10c	[30,70]	[75,100]	8	243.16	90	76.77	14.58	21.88	39.93	16.44	3,18	0.8157	2.78
DP-MS	50u	Yes	10c	[30,70]	[75,100]	8	237.84	90	74.09	14.58	24.27	34.90	49.97	3,16	0.8177	3.13
DP-CG	50u	Yes	10c	[30,70]	[75,100]	8	237.42	90	73.37	13.96	23.54	36.55	52.29	3,16	0.8017	3.13
ST	50u	Yes	10c	[30,70]	[75,100]	9	466.91	210	165.97	15.00	60.00	15.94	10.88	7,7	0.7421	7.86
MT	50u	Yes	10c	[30,70]	[75,100]	9	310.62	120	110.82	15.00	26.13	38.68	17.65	4,17	0.8111	3.24
DP-MS	50u	Yes	10c	[30,70]	[75,100]	9	307.17	120	107.37	15.73	26.09	37.97	64.18	4,18	0.7983	3.06
DP-CG	50u	Yes	10c	[30,70]	[75,100]	9	305.61	120	105.44	14.48	24.53	41.16	59.85	4,18	0.7710	3.06
ST	50u	Yes	1c	[80,120]	[150,175]	0	858.23	510	258.54	22.67	63.38	3.65	15.53	17,17	0.4307	5.06
MT	50u	Yes	1c	[80,120]	[150,175]	0	487.94	210	216.46	22.72	30.34	8.41	80.21	7,39	0.8080	2.21
DP-MS	50u	Yes	1c	[80,120]	[150,175]	0	495.55	210	224.57	22.33	30.25	8.40	220.03	7,38	0.8294	2.26
DP-CG	50u	Yes	1c	[80,120]	[150,175]	0	491.69	210	220.53	22.13	30.78	8.26	301.37	7,37	0.8190	2.32
ST	50u	Yes	1c	[80,120]	[150,175]	1	1232.53	660	426.41	33.58	107.46	5.09	232.04	22,22	0.5479	5.45
MT	50u	Yes	1c	[80,120]	[150,175]	1	753.31	330	332.78	35.15	41.79	13.59	498.44	11,60	0.7799	2.00
DP-MS	50u	Yes	1c	[80,120]	[150,175]	1	736.01	300	344.92	34.83	42.96	13.29	790.10	10,60	0.8843	2.00
DP-CG	50u	Yes	1c	[80,120]	[150,175]	1	722.84	300	331.41	34.61	42.84	13.99	754.62	10,62	0.8564	1.94
ST	50u	Yes	1c	[80,120]	[150,175]	2	1222.20	660	420.79	34.11	102.23	5.07	216.80	22,22	0.5375	5.27
MT	50u	Yes	1c	[80,120]	[150,175]	2	724.65	300	332.29	35.51	43.75	13.09	262.76	10,59	0.8627	1.97
DP-MS	50u	Yes	1c	[80,120]	[150,175]	2	718.95	300	326.30	34.98	44.95	12.73	705.53	10,57	0.8524	2.04
DP-CG	50u	Yes	1c	[80,120]	[150,175]	2	724.17	300	330.78	36.25	44.42	12.73	815.81	10,58	0.8632	2.00
ST	50u	Yes	1c	[80,120]	[150,175]	3	849.03	510	250.23	22.79	62.10	3.90	181.85	17,17	0.4192	4.94
MT	50u	Yes	1c	[80,120]	[150,175]	3	499.42	210	223.71	25.04	32.46	8.21	85.31	7,39	0.8445	2.15
DP-MS	50u	Yes	1c	[80,120]	[150,175]	3	485.70	210	210.11	24.16	32.72	8.70	255.38	7,40	0.8035	2.10
DP-CG	50u	Yes	1c	[80,120]	[150,175]	3	490.50	210	215.61	24.94	31.13	8.83	258.23	7,41	0.8162	2.05
ST	50u	Yes	1c	[80,120]	[150,175]	4	940.99	510	327.95	24.17	74.90	3.97	206.28	17,17	0.5315	5.53
MT	50u	Yes	1c	[80,120]	[150,175]	4	621.84	270	281.90	26.32	34.10	9.53	113.70	9,43	0.7943	2.19
DP-MS	50u	Yes	1c	[80,120]	[150,175]	4	624.19	270	284.85	25.82	34.17	9.35	266.09	9,42	0.7996	2.24
DP-CG	50u	Yes	1c	[80,120]	[150,175]	4	630.65	270	292.77	25.46	33.10	9.31	390.73	9,41	0.8133	2.29
ST	50u	Yes	1c	[80,120]	[150,175]	5	1289.31	720	434.95	32.07	96.77	5.52	219.13	24,24	0.4967	4.92
MT	50u	Yes	1c	[80,120]	[150,175]	5	755.78	330	336.58	33.04	44.73	11.43	300.56	11,51	0.7887	2.31
DP-MS	50u	Yes	1c	[80,120]	[150,175]	5	722.21	300	335.01	31.83	43.30	12.07	659.23	10,53	0.8579	2.23
DP-CG	50u	Yes	1c	[80,120]	[150,175]	5	722.12	300	333.98	32.58	43.99	11.57	764.77	10,52	0.8594	2.27
ST	50u	Yes	1c	[80,120]	[150,175]	6	1188.00	690	371.93	30.92	89.75	5.41	230.10	23,23	0.4546	5.04
MT	50u	Yes	1c	[80,120]	[150,175]	6	694.06	300	308.90	32.31	40.58	12.26	356.38	10,54	0.8000	2.15
DP-MS	50u	Yes	1c	[80,120]	[150,175]	6	674.82	270	315.87	33.56	43.67	11.72	685.83	9,53	0.9165	2.19
DP-CG	50u	Yes	1c	[80,120]	[150,175]	6	665.19	270	308.99	32.79	41.40	12.01	671.22	9,54	0.8927	2.15
ST	50u	Yes	1c	[80,120]	[150,175]	7	1131.53	630	375.27	31.04	90.31	4.90	227.42	21,21	0.5019	5.14
MT	50u	Yes	1c	[80,120]	[150,175]	7	672.54	270	315.36	32.71	43.40	11.07	187.22	9,50	0.9122	2.16
DP-MS	50u	Yes	1c	[80,120]	[150,175]	7	658.33	270	302.22	31.83	43.42	10.86	548.36	9,48	0.8806	2.25
DP-CG	50u	Yes	1c	[80,120]	[150,175]	7	644.80	270	289.09	31.96	42.03	11.72	652.73	9,52	0.8480	2.08
ST	50u	Yes	1c	[80,120]	[150,175]	8	1166.94	630	414.43	29.46	88.15	4.90	219.77	21,21	0.5347	5.33
MT	50u	Yes	1c	[80,120]	[150,175]	8	691.09	300	307.46	30.63	41.99	11.02	242.29	10,49	0.7964	2.29
DP-MS	50u	Yes	1c	[80,120]	[150,175]	8	690.44	300	307.28	30.83	41.11	11.23	511.22	10,50	0.7944	2.24
DP-CG	50u	Yes	1c	[80,120]	[150,175]	8	662.37	270	309.40	30.84	40.56	11.57	565.42	9,51	0.8859	2.20
ST	50u	Yes	1c	[80,120]	[150,175]	9	1210.73	720	381.74	29.25	74.15	5.59	115.35	24,24	0.4258	4.29
MT	50u	Yes	1c	[80,120]	[150,175]	9	704.01	300	326.73	31.58	33.10	12.59	161.22	10,57	0.8152	1.81
DP-MS	50u	Yes	1c	[80,120]	[150,175]	9	709.74	300	332.21	31.08	34.50	11.95	450.67	10,54	0.8284	1.91
DP-CG	50u	Yes	1c	[80,120]	[150,175]	9	700.26	300	324.67	30.09	32.84	12.67	543.96	10,56	0.8066	1.84
ST	50u	Yes	1c	[80,120]	[75,100]	0	649.13	300	227.98	22.50	96.25	2.40	35.48	10,10	0.7528	8.30
MT	50u	Yes	1c	[80,120]	[75,100]	0	397.36	180	153.72	24.42	29.34	9.88	121.18	6,44	0.7364	1.89
DP-MS	50u	Yes	1c	[80,120]	[75,100]	0	367.79	150	153.54	24.27	31.41	8.58	440.14	5,39	0.8925	2.13
DP-CG	50u	Yes	1c	[80,120]	[75,100]	0	362.42	150	149.76	23.77	29.21	9.68	465.01	5,43	0.8639	1.93
ST	50u	Yes	1c	[80,120]	[75,100]	1	821.48	390	273.35	28.75	126.25	3.13	203.33	13,13	0.7186	8.46
MT	50u	Yes	1c	[80,120]	[75,100]	1	478.74	210	185.50	30.17	42.31	10.77	254.74	7,47	0.7888	2.34
DP-MS	50u	Yes	1c	[80,120]	[75,100]	1	474.71	210	180.18	30.31	44.22	10.01	831.73	7,44	0.7810	2.50
DP-CG	50u	Yes	1c	[80,120]	[75,100]	1	468.06	210	176.09	30.42	39.66	11.88	920.71	7,53	0.7534	2.08
ST	50u	Yes	1c	[80,120]	[75,100]	2	825.62	390	253.76	32.50	146.25	3.11	93.06	13,13	0.7341	9.00
MT	50u	Yes	1c	[80,120]	[75,100]	2	457.86	210	160.05	33.48	40.52	13.80	422.16	7,60	0.7216	1.95
DP-MS	50u	Yes	1c	[80,120]	[75,100]	2	429.89	180	161.93	33.71	41.58	12.67	1434.16	6,55	0.8535	2.13
DP-CG	50u	Yes	1c	[80,120]	[75,100]	2	433.63	180	165.41	34.13	41.36	12.73	1529.80	6,56	0.8659	2.09
ST	50u	Yes	1c	[80,120]	[75,100]	3	581.55	300	167.93	21.25	90.00	2.38	52.47	10,10	0.6140	8.10
MT	50u	Yes	1c	[80,120]	[75,100]	3	290.17	120	110.50	21.83	29.58	8.25	101.73	4,36	0.8739	2.25
DP-MS	50u	Yes	1c	[80,120]	[75,100]	3	286.54	120	108.29	22.46	27.26	8.53	493.25	4,38	0.8522	2.13
DP-CG	50u	Yes	1c	[80,120]	[75,100]	3	282.22	120	103.67	22.21	27.36	8.98	357.57	4,40	0.8282	2.03
ST	50u	Yes	1c	[80,120]	[75,100]	4	735.85	360	243.08	24.75	105.13	2.89	67.31	12,12	0.6757	7.75
MT	50u	Yes	1c	[80,120]	[75,100]	4	417.91	180	168.27	25.92	33.50	10.23	163.29	6,45	0.8085	2.07
DP-MS	50u	Yes	1c	[80,120]	[75,100]	4	414.05	180	163.48	26.60	33.15	10.82	607.01	6,48	0.7939	1.94
DP-CG	50u	Yes	1c	[80,120]	[75,100]	4	408.09	180	159.10	26.27	31.61	11.10	593.71	6,49	0.7715	1.90
ST	50u	Yes	1c	[80,120]	[75,100]	5	804.03	390	237.17	31.25	142.50	3.11	73.29	13,13	0.6990	9.15
MT	50u	Yes	1c	[80,120]	[75,100]	5	426.29	180	156.88	32.63	44.90	11.88	339.20	6,52	0.8459	2.29
DP-MS	50u	Yes	1c	[80,120]	[75,100]	5	422.30	180	154.75	32.46	42.15	12.95	1494.57	6,57	0.8267	2.09
DP-CG	50u	Yes	1c	[80,120]	[75,100]	5	421.63	180	153.24	33.23	42.36	12.80	1695.46	6,57	0.8258	2.09
ST	50u	Yes	1c	[80,120]	[75,100]	6	708.40	360	209.89	26.25	109.38	2.89	113.35	12,12	0.6324	8.17
MT	50u	Yes	1c	[80,120]	[75,100]	6	406.70	180	150.36	27.67	39.06	9.61	150.17	6,42	0.7792	2.33
DP-MS	50u	Yes	1c	[80,120]	[75,100]	6	405.31	180	150.54	27.90	36.53	10.33	676.63	6,46	0.7703	2.13
DP-CG	50u	Yes	1c	[80,120]	[75,100]	6	403.72	180	148.94	28.25	35.85	10.69	927.51	6,48	0.7635	2.04
ST	50u	Yes	1c	[80,120]	[75,100]	7	802.92	390	257.30	28.75	123.75	3.11	229.84	13,13	0.6891	8.31
MT	50u	Yes	1c	[80,120]	[75,100]	7	479.75	210	187.50	30.46	41.54	10.26	249.43	7,45	0.7928	2.40
DP-MS	50u	Yes	1c	[80,120]	[75,100]	7	484.56	210	190.86	29.25	44.56	9.90	701.84	7,43	0.8089	2.51
DP-CG	50u	Yes	1c	[80,120]	[75,100]	7	479.91	210	187.85	29.73	41.67	10.67	781.00	7,47	0.7917	2.30
ST	50u	Yes	1c	[80,120]	[75,100]	8	675.50	330	224.07	23.33	95.42	2.68	53.19	11,11	0.6773	7.73
MT	50u	Yes	1c	[80,120]	[75,100]	8	367.57	150	150.36	25.50	32.83	8.87	128.22	5,40	0.8931	2.13
DP-MS	50u	Yes	1c	[80,120]	[75,100]	8	374.41	150	158.49	24.65	32.16	9.12	418.29	5,40	0.9180	2.13
DP-CG	50u	Yes	1c	[80,120]	[75,100]	8	369.54	150	152.10	25.13	33.01	9.29	504.23	5,42	0.8991	2.02
ST	50u	Yes	1c	[80,120]	[75,100]	9	612.60	300	192.31	22.08	95.83	2.37	25.22	10,10	0.6794	8.60
MT	50u	Yes	1c	[80,120]	[75,100]	9	344.62	150	133.54	22.29	30.44	8.34	136.83	5,36	0.7978	2.39
DP-MS	50u	Yes	1c	[80,120]	[75,100]	9	344.62	150	133.96	22.19	30.05	8.42	376.26	5,36	0.7970	2.39
DP-CG	50u	Yes	1c	[80,120]	[75,100]	9	344.47	150	132.50	22.96	30.75	8.26	439.54	5,37	0.7985	2.32

ST	50u	Yes	1c	[30,70]	[150,175]	0	675.22	390	225.20	15.23	41.94	2.84	2.44	13,13	0.4564	4.38
MT	50u	Yes	1c	[30,70]	[150,175]	0	399.10	180	175.72	16.39	21.72	5.26	33.54	6,25	0.7445	2.28
DP-MS	50u	Yes	1c	[30,70]	[150,175]	0	377.54	150	184.07	16.63	21.75	5.09	59.53	5,24	0.9282	2.38
DP-CG	50u	Yes	1c	[30,70]	[150,175]	0	394.52	180	172.08	16.94	19.53	5.97	84.86	6,29	0.7256	1.97
ST	50u	Yes	1c	[30,70]	[150,175]	1	364.55	210	120.11	8.75	24.06	1.62	0.54	7,7	0.4604	4.71
MT	50u	Yes	1c	[30,70]	[150,175]	1	251.70	120	105.60	10.08	12.67	3.35	3.46	4,16	0.6702	2.06
DP-MS	50u	Yes	1c	[30,70]	[150,175]	1	251.69	120	105.72	10.27	12.13	3.57	11.20	4,17	0.6686	1.94
DP-CG	50u	Yes	1c	[30,70]	[150,175]	1	246.92	120	100.79	9.46	13.30	3.37	17.40	4,15	0.6462	2.20
ST	50u	Yes	1c	[30,70]	[150,175]	2	703.60	420	223.79	15.71	40.81	3.29	15.96	14,14	0.4206	4.43
MT	50u	Yes	1c	[30,70]	[150,175]	2	419.26	180	193.15	16.36	23.82	5.94	26.04	6,27	0.8112	2.30
DP-MS	50u	Yes	1c	[30,70]	[150,175]	2	411.13	180	186.71	16.63	21.33	6.47	68.33	6,29	0.7805	2.14
DP-CG	50u	Yes	1c	[30,70]	[150,175]	2	409.68	180	185.33	16.04	22.15	6.16	89.14	6,27	0.7769	2.30
ST	50u	Yes	1c	[30,70]	[150,175]	3	517.08	300	173.09	12.40	29.48	2.11	0.63	10,10	0.4509	3.80
MT	50u	Yes	1c	[30,70]	[150,175]	3	327.26	150	145.08	13.39	14.97	3.81	3.54	5,19	0.7221	2.00
DP-MS	50u	Yes	1c	[30,70]	[150,175]	3	320.86	150	139.13	13.36	14.02	4.35	15.25	5,22	0.6934	1.73
DP-CG	50u	Yes	1c	[30,70]	[150,175]	3	299.45	120	147.52	13.65	14.22	4.06	15.47	4,21	0.9117	1.81
ST	50u	Yes	1c	[30,70]	[150,175]	4	587.31	330	200.01	15.38	39.44	2.48	1.58	11,11	0.4882	4.27
MT	50u	Yes	1c	[30,70]	[150,175]	4	359.94	150	170.38	15.75	18.75	5.07	16.64	5,23	0.8540	2.04
DP-MS	50u	Yes	1c	[30,70]	[150,175]	4	357.36	150	167.81	15.58	18.98	4.98	27.78	5,22	0.8441	2.14
DP-CG	50u	Yes	1c	[30,70]	[150,175]	4	352.68	150	162.58	15.77	19.07	5.26	28.03	5,24	0.8245	1.96
ST	50u	Yes	1c	[30,70]	[150,175]	5	635.40	360	215.42	16.75	40.50	2.72	2.74	12,12	0.4783	4.25
MT	50u	Yes	1c	[30,70]	[150,175]	5	390.25	180	167.62	17.14	19.60	5.89	13.90	6,27	0.7118	1.89
DP-MS	50u	Yes	1c	[30,70]	[150,175]	5	371.76	150	178.36	17.27	20.42	5.71	46.53	5,26	0.9019	1.96
DP-CG	50u	Yes	1c	[30,70]	[150,175]	5	366.79	150	173.78	17.67	19.02	6.32	44.49	5,29	0.8785	1.76
ST	50u	Yes	1c	[30,70]	[150,175]	6	450.69	270	142.77	10.63	25.21	2.09	0.89	9,9	0.4168	4.33
MT	50u	Yes	1c	[30,70]	[150,175]	6	261.87	120	112.47	11.88	13.42	4.11	5.06	4,19	0.7204	2.05
DP-MS	50u	Yes	1c	[30,70]	[150,175]	6	260.77	120	111.83	12.31	12.20	4.43	22.14	4,21	0.7124	1.86
DP-CG	50u	Yes	1c	[30,70]	[150,175]	6	259.84	120	110.59	12.06	12.89	4.30	21.91	4,20	0.7089	1.95
ST	50u	Yes	1c	[30,70]	[150,175]	7	325.84	180	115.31	7.92	21.25	1.36	0.61	6,6	0.5059	5.17
MT	50u	Yes	1c	[30,70]	[150,175]	7	199.62	90	87.58	7.83	11.08	3.12	2.78	3,14	0.7415	2.21
DP-MS	50u	Yes	1c	[30,70]	[150,175]	7	201.63	90	88.24	7.81	12.97	2.62	10.60	3,12	0.7614	2.58
DP-CG	50u	Yes	1c	[30,70]	[150,175]	7	200.84	90	88.07	7.60	12.45	2.72	9.89	3,12	0.7542	2.58
ST	50u	Yes	1c	[30,70]	[150,175]	8	471.27	270	158.37	11.75	29.13	2.03	0.92	9,9	0.4655	4.78
MT	50u	Yes	1c	[30,70]	[150,175]	8	322.16	150	137.27	12.59	18.97	3.33	5.28	5,16	0.7069	2.69
DP-MS	50u	Yes	1c	[30,70]	[150,175]	8	279.11	120	126.36	12.33	16.27	4.15	21.78	4,19	0.8106	2.26
DP-CG	50u	Yes	1c	[30,70]	[150,175]	8	291.15	120	137.79	12.42	16.71	4.23	24.42	4,20	0.8710	2.15
ST	50u	Yes	1c	[30,70]	[150,175]	9	546.61	300	191.22	14.38	38.75	2.27	2.52	10,10	0.5152	4.70
MT	50u	Yes	1c	[30,70]	[150,175]	9	348.11	150	157.29	15.25	21.29	4.28	9.29	5,20	0.8119	2.35
DP-MS	50u	Yes	1c	[30,70]	[150,175]	9	337.30	150	147.75	15.69	18.32	5.54	32.69	5,26	0.7610	1.81
DP-CG	50u	Yes	1c	[30,70]	[150,175]	9	338.18	150	148.86	15.46	18.46	5.40	38.69	5,25	0.7650	1.88
ST	50u	Yes	1c	[30,70]	[75,100]	0	260.58	120	95.29	8.75	35.63	0.92	0.72	4,4	0.7538	7.75
MT	50u	Yes	1c	[30,70]	[75,100]	0	170.82	90	56.30	9.38	11.98	3.16	3.42	3,15	0.5533	2.07
DP-MS	50u	Yes	1c	[30,70]	[75,100]	0	141.45	60	56.86	9.38	12.03	3.19	16.71	2,15	0.8362	2.07
DP-CG	50u	Yes	1c	[30,70]	[75,100]	0	142.81	60	57.74	9.69	12.34	3.04	18.05	2,15	0.8528	2.07
ST	50u	Yes	1c	[30,70]	[75,100]	1	372.73	180	119.20	13.33	58.75	1.45	2.64	6,6	0.6977	8.50
MT	50u	Yes	1c	[30,70]	[75,100]	1	206.26	90	78.37	14.38	18.56	4.95	15.23	3,22	0.7970	2.32
DP-MS	50u	Yes	1c	[30,70]	[75,100]	1	198.80	90	71.20	14.46	17.77	5.36	90.02	3,24	0.7433	2.13
DP-CG	50u	Yes	1c	[30,70]	[75,100]	1	200.21	90	71.58	14.29	19.56	4.78	110.17	3,21	0.7593	2.43
ST	50u	Yes	1c	[30,70]	[75,100]	2	348.38	180	114.82	10.42	41.67	1.48	1.55	6,6	0.5997	7.17
MT	50u	Yes	1c	[30,70]	[75,100]	2	189.32	90	67.79	11.38	16.38	3.78	15.36	3,17	0.6832	2.53
DP-MS	50u	Yes	1c	[30,70]	[75,100]	2	184.73	90	64.90	11.13	14.11	4.59	51.60	3,20	0.6430	2.15
DP-CG	50u	Yes	1c	[30,70]	[75,100]	2	187.15	90	67.37	10.98	14.14	4.66	49.11	3,20	0.6584	2.15
ST	50u	Yes	1c	[30,70]	[75,100]	3	418.35	210	137.18	14.25	55.25	1.67	9.73	7,7	0.6402	7.43
MT	50u	Yes	1c	[30,70]	[75,100]	3	261.34	120	99.71	15.21	21.77	4.65	14.73	4,21	0.7297	2.48
DP-MS	50u	Yes	1c	[30,70]	[75,100]	3	256.89	120	96.31	14.29	21.35	4.94	71.47	4,21	0.7043	2.48
DP-CG	50u	Yes	1c	[30,70]	[75,100]	3	255.61	120	96.38	14.67	18.79	5.77	111.57	4,25	0.6910	2.08
ST	50u	Yes	1c	[30,70]	[75,100]	4	345.78	180	108.72	11.25	44.38	1.43	3.16	6,6	0.5942	7.50
MT	50u	Yes	1c	[30,70]	[75,100]	4	193.02	90	69.85	11.88	17.29	4.00	23.72	3,18	0.7087	2.50
DP-MS	50u	Yes	1c	[30,70]	[75,100]	4	191.85	90	68.92	11.96	16.63	4.35	55.18	3,20	0.6977	2.25
DP-CG	50u	Yes	1c	[30,70]	[75,100]	4	191.80	90	69.15	11.79	16.55	4.30	62.07	3,19	0.6972	2.37
ST	50u	Yes	1c	[30,70]	[75,100]	5	540.31	270	171.31	20.00	76.88	2.12	17.22	9,9	0.6498	7.44
MT	50u	Yes	1c	[30,70]	[75,100]	5	324.45	150	119.98	21.38	25.38	7.72	48.11	5,36	0.7137	1.86
DP-MS	50u	Yes	1c	[30,70]	[75,100]	5	296.90	120	122.48	20.44	26.32	7.67	220.42	4,34	0.9046	1.97
DP-CG	50u	Yes	1c	[30,70]	[75,100]	5	295.15	120	121.84	21.04	24.38	7.89	259.63	4,36	0.8931	1.86
ST	50u	Yes	1c	[30,70]	[75,100]	6	441.42	210	147.27	15.00	67.50	1.64	10.92	7,7	0.7154	8.86
MT	50u	Yes	1c	[30,70]	[75,100]	6	254.34	120	89.07	16.00	24.06	5.21	43.41	4,24	0.6958	2.58
DP-MS	50u	Yes	1c	[30,70]	[75,100]	6	247.33	120	84.36	15.39	21.35	6.23	140.02	4,28	0.6514	2.21
DP-CG	50u	Yes	1c	[30,70]	[75,100]	6	226.01	90	91.20	16.04	23.33	5.43	153.37	3,25	0.9362	2.48
ST	50u	Yes	1c	[30,70]	[75,100]	7	234.36	120	78.60	7.08	27.71	0.97	0.52	4,4	0.6105	7.50
MT	50u	Yes	1c	[30,70]	[75,100]	7	138.69	60	55.63	8.75	11.63	2.69	4.14	2,13	0.8110	2.31
DP-MS	50u	Yes	1c	[30,70]	[75,100]	7	133.03	60	50.81	8.96	10.63	2.63	15.92	2,13	0.7529	2.31
DP-CG	50u	Yes	1c	[30,70]	[75,100]	7	130.97	60	49.76	7.60	11.09	2.51	15.16	2,12	0.7314	2.50
ST	50u	Yes	1c	[30,70]	[75,100]	8	364.77	180	112.10	13.75	57.50	1.42	12.16	6,6	0.6706	8.33
MT	50u	Yes	1c	[30,70]	[75,100]	8	209.56	90	79.96	14.08	21.13	4.40	14.71	3,19	0.8264	2.63
DP-MS	50u	Yes	1c	[30,70]	[75,100]	8	200.49	90	71.07	14.25	20.41	4.76	69.75	3,21	0.7626	2.38
DP-CG	50u	Yes	1c	[30,70]	[75,100]	8	203.04	90	74.48	14.13	19.44	4.99	88.26	3,22	0.7762	2.27
ST	50u	Yes	1c	[30,70]	[75,100]	9	457.13	240	148.09	14.25	52.88	1.91	12.02	8,8	0.5800	6.88
MT	50u	Yes	1c	[30,70]	[75,100]	9	272.68	120	109.69	15.63	22.29	5.07	24.35	4,23	0.7855	2.39
DP-MS	50u	Yes	1c	[30,70]	[75,100]	9	261.89	120	100.84	15.15	20.05	5.86	89.72	4,26	0.7242	2.12
DP-CG	50u	Yes	1c	[30,70]	[75,100]	9	266.00	120	104.52	15.17	20.63	5.69	103.38	4,25	0.7463	2.20